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Mapping GDP-linked bonds: the case of the Greek economy

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Abstract

We develop and present a simplified model, alternative to the pricing of GDP-linked bonds. Its main philosophy is to map these specialized debt products to normal fixed income securities. This process makes them better understood and attractive to market participants. The key feature of GDP-linked bonds is that their coupons are related to the economic growth of the issuers. As a result, designing and mapping these instruments requires macroeconomic modelling of basic economic growth parameters, such as the output gap of the issuing country. This is done in this article for the case of Greece, and mapping the various indexed bond products is performed through Monte Carlo simulations. The model can also be applied to other countries, provided that the data is adjusted. The pricing of these products, incorporating also default issues, is a complicated matter.

Keywords: *GDP-linked bonds, plain vanilla bonds.*

JEL classification: *G12, G38.*

1. Introduction

The term GDP-linked bonds refers to bonds whose repayments are not fixed, but are indexed to the country's economic situation and, more specifically, to the Gross Domestic Product (GDP). The idea of issuing this kind of debt instruments is not new. However, in recent years, a significant increase in debt levels has been witnessed in advanced and emerging economies around the world. The main reason seems to be the economic recession, which affected the debt/GDP ratios for the majority of countries. As a result, the idea of replacing conventional methods with instruments related to the country's economic situation becomes more and more accepted and the dynamics of GDP-linked bonds become stronger.

Several influential academics have made a strong case for issuing GDP-linked bonds over the past two and a half decades, for instance Shiller (1993), Borensztein et al. (2004), Blanchart, Mauro & Acalin (2016). Their message is gradually filtering into policy circles. Some central banks, such as the Bank of England, have been active in exploring the potential for such instruments (see Benford, Best & Joy 2016). International organisations, including recently the IMF, have provided a nuanced, but overall supportive assessment of the prospects for state-contingent debt instruments (International Monetary Fund [IMF] 2017). As a sign of rising interest, policy makers of the G20 (2017) countries held a discussion on a "compass" for GDP-linked bonds.

Debt instruments related to a country's output are widely considered as stabilizers for debt sustainability. Indeed, in this case, payments are automatically related to the sovereign's ability to pay. Consequently, according to Abbas et al. (2019), when revenues are weak and the economy faces some downturn, the payments fall significantly. Instead, in conventional bonds, payment remains stable, imposing further constraints in the economy and leading to a vicious cycle. Thus, from this point of view, GDP-linked bonds are an attractive tool, since they allow fiscal space during

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recessions and reduce the probability of default, which is extremely costly for both issuers and bondholders.

The common view on GDP-linked bonds is (Abbas et al. (2019)) that they combine attractive properties in principle with practical challenges that explain their limited take-up so far. GDP-linked bonds offer an in-built mechanism to stabilise public debts in an environment of uncertainty. This is especially valuable where indebtedness is at sustainable, but high levels, and winding down debt through conventional adjustment takes time. Investors, on their side, are interested in a reduction of the risk of outright default. In addition, some investors may find appealing an instrument guaranteeing returns linked to average growth, given their needs and the nature of their liabilities. However, GDP-linked bonds have not been used so far in advanced economies, and their occasional use in debt restructuring contributes to a perception that GDP-linked bonds are an instrument for debt-distressed countries only.

This can be attributed to the problems associated with GDP-indexed bonds. First, GDP-linked bonds introduce the temptation by the issuing country to alter the GDP statistics to reduce payments. In addition, a country that knows that its growth determines its payments may seek to grow less. These two factors add to the risk premium imposed on the GDP-indexed bonds, increasing the cost and reducing the savings associated with them. An even more important problem with GDP-indexed bonds is their financial structure. A GDP-indexed bond can be designed in many ways. The more complex the design, the more difficult to price and the lower the value.

By reducing the debt service in bad times (and the reverse), GDP growth risk is transferred from the debtor country to the creditor, who, in turn, needs to hedge it, or pool it under some particular investment benchmark. The problem is that growth risk is exotic, difficult to price and hedge properly (no financial instrument has a high correlation to real growth) and, therefore, costly for investors. As a result, the investor universe for these bonds is narrow and highly speculative, which, again, adds to the cost of the debt.

However, there have been a few experiences, mainly in the context of debt restructurings, with instruments sharing some features of GDP-linked bonds and primarily taking the shape of non-tradable instruments (International Monetary Fund [IMF] 2017). No sovereign has yet issued a GDP-linked bond with complete and symmetric risk sharing between sovereigns and private investors –falling with lower GDP and rising with higher GDP (see Benford, Best & Joy (2016), OECD (2017)). The most successful experience of state-contingent debt securities is inflation-linked bonds, which were first launched in the 1980s. Today, these instruments represent 7% of government debt overall in OECD countries (OECD, 2017), but this share reaches a quarter of government debt in the UK, and above 10% in Italy and France.

Current times may slowly become more propitious (Abbas et al. (2019)) to the notion of GDP-linked bonds, including in advanced economies and, notably, European countries. The economic conditions and the fiscal outlook have recently improved across EU countries, in liaison with a broadening recovery. Nevertheless, public debt levels are high as a legacy of the crisis, while long-term growth prospects remain uncertain. Policy makers must guarantee debt sustainability in that environment. GDP-linked bonds can efficiently contribute to that objective by reducing risks of disruptive crises through smooth adjustments. GDP-linked bonds may also increase short-run budget flexibility and cross-border risk sharing, especially when a significant share of debt is foreign owned. These benefits, in terms of long-run solvency and resilience to shocks, have a particular relevance for euro area countries. In addition, GDP-linked bonds may be an indirect, but effective means to incentivise economic reforms. In sum, in the post-crisis environment, the case for GDP-linked bonds may be stronger than before, motivating further analysis of their implications.

As mentioned above, the familiarity of markets, investors, sovereigns, etc. with this type of debt instrument is crucial for its further development and acceptance. Towards this direction, the first, and probably the biggest, issue that has to be addressed is the pricing of these bonds. While in the conventional bonds where things are clear and easily understood by the majority of the participants, the pricing of GDP-linked bonds is a sophisticated matter. To date, few attempts have been made for developing a pricing framework for these bonds (see Borensztein et al. (2004), Chamon & Mauro (2006), Ruban, Poon & Vonatsos (2008), Shiller (1993), Shiller, Ostry and Benford, eds. (2018).

Borensztein et al. (2004) simulate the effects of GDP-indexed bonds under different assumptions about fiscal policy reaction functions and their output effects and find that they could substantially reduce the likelihood of debt/GDP paths becoming explosive. Chamon & Mauro (2006) present a different way of pricing growth-indexed bonds. They start with the observation that there is a tendency of emerging markets to be “debt intolerant” and to default when their debt/GDP ratio exceeds a certain value. As a result, their approach is based on a default-trigger rule for the debt/GDP ratio. Ruban, Poon & Vonatsos (2008) obtain prices and default profiles

for vanilla bonds and various GDP-linked structures that could be issued by emerging market sovereigns. Shiller (1993), Shiller, Ostry and Benford, eds. (2018) present a collection of articles on the rationale and design of GDP-linked bonds.

In this article, we present a model for pricing GDP-linked bonds and apply it to the case of the Greek economy. The GDP of the sovereign is modelled appropriately as the superposition of a trend and a cyclical component which follows the Ornstein-Uhlenbeck process. In order to make the indexation of the GDP-linked bonds more transparent, we map them to plain vanilla bonds. We use historical data of the Greek economy over the past 60 years and decompose the Greek GDP, according to the proposed model, into a trend and a cyclical component. We use this decomposition to index GDP-linked bonds by using various pricing scenarios. Our study can be used in principle to price GDP-linked bonds for any economy should the economic data be properly adjusted. In the course of our study, another interesting future of GDP-linked bonds emerges: The curves giving to these bonds the corresponding yield of a plain vanilla bond contain some important information about the economy of the country issuing the GDP-linked bonds (about Greece in the case under consideration). The specifics are given in Section 5.

This article is organized as follows: In Section 2, we develop a model for the GDP of the sovereign. In Section 3, we analyze the correspondence between a GDP-linked bond and a plain vanilla bond. In Section 4, we use historical data for Greece over the past 60 years and decompose the Greek GDP time series over these years into a trend and a cyclical component. In Section 5, we apply several pricing scenarios of a GDP-linked bond to the case of Greek economy. In Section 6, we make a few comments regarding the implementation of a GDP-linked bond. In Section 7, we conclude the article and outline future directions for research. Finally, in the Appendix, we give the bare essentials of the Hodrick-Prescott filter, which isolates the cyclical component of an economic time series.

2. Model description

In this section, we develop a simple structural model for GDP-linked bonds that relates debt cash flows to the dynamics of macroeconomic variables.

For simplicity, we will assume that all the market debt of the sovereign is represented by one bond, denominated in foreign currency, and that we are concerned with valuing this bond. After initial time $t = 0$, there is no further issuance of debt. Our next assumption is that the sovereign's assets that can be devoted to debt service at time t are a function of the potential output (or output trend) at that time.

Potential output is typically defined as a measure of sustainable output in the economy, in which the intensity of resource use is not adding to or reducing inflationary pressure. It is also a measure of the real GDP trend, where the influence of short-term shocks and the business cycle has been removed.

The motivation behind this assumption is that higher sustainable growth makes it possible to accumulate greater resources to service debt. It can also be related to the additional borrowing capacity available to the sovereign, should new debt be used to service old debt in order to avoid default. We denote potential real output (real GDP trend) at time t in domestic currency by $\overline{Y}_t$.

Actual real GDP in domestic currency at time t, which we denote by Y_t, is related to potential GDP through the output gap. The output gap at time t, G_t, is defined as the ratio of actual to potential output, so that

$$\overline{Y}_t = G_t Y_t. \tag{2.1}$$

We assume that $\overline{Y}_t$ satisfies

$$d\overline{Y}_t = \mu \overline{Y}_t dt. \tag{2.2}$$

Intuitively, μ determines the expected long-run sustainable growth rate.

By taking logarithms at both sides of (2.1), we obtain

$$\ln\overline{Y}_t = \ln G_t + \ln Y_t.$$

We assume that the logarithm of G_t, hereafter denoted by g_t, follows an Ornstein-Uhlenbeck process

$$dg_t = -kg_t dt + VdW_t \tag{2.3}$$

where $k > 0$, $V > 0$ are constants and W_t denotes the Wiener process. In economic terms, V drives the severity of the business cycle, or temporary booms and contractions, while k determines its length.

The Ornstein-Uhlenbeck process is a temporally homogeneous Gauss-Markov process. Temporally homogeneous means that the probability distributions are unaffected by translations of the t-axis. In fact, it is the only nontrivial process that satisfies these three conditions, up to allowing linear transformations of the space and time variables. More importantly, it is a mean-reverting process, which means that it tends to drift towards its mean function over time. It is precisely this last property that makes it appropriate to model GDP because its fluctuations in the vicinity of the trend

Y_t are driven, even by the sovereign policy if this is necessary, towards their mean value.

Implementation of the model described in Section 2 faces the same basic issue that Burns and Mitchell (1946) did fifty years ago: How should one isolate the cyclical component of an economic time series? In particular, how should one separate business-cycle elements from slowly evolving secular trends and rapidly varying seasonal or irregular components? The solution of the problem involves defining what one means by the business cycle and transforming or filtering the macroeconomic series to coincide with this definition. In Appendix A, we revise the so-called Hodrick-Prescott filter for decomposing a time series into growth and cyclical components.

We next use model (2.1), (2.2) and (2.3), in order to index bonds linked to the GDP of Greece. The first step towards this goal is to map GDP-linked bonds with the plain vanilla bonds.

3. Correspondence of GDP-linked bonds with the plain vanilla bonds

The characteristic features of a growth-indexed bond are not always easy to understand. As mentioned in the introduction, this is one of the reasons this kind of bond is not very common. It has also been suggested that the indexation of these bonds should take as simple a form as possible. In this section, we develop a methodology for analyzing the correspondence between a GDP-linked bond and a plain vanilla bond. The expectation is that both issuers and investors would have a better understanding of the value of the bond and gain some familiarity with this instrument.

We recall that plain vanilla bonds are the most basic and standard version of bonds. They have the following characteristic features: (a) fixed coupon rate, i.e., the payment obligations of the issuer are standard, (b) fixed and predetermined time of coupon payment, (c) fixed and predetermined date of maturity and (d) fixed face value of the bond. Hence, plain vanilla bonds are easily understood by both issuers and investors. Consequently, establishing a correspondence between plain vanilla bonds and GDP-linked bonds is expected to improve our understanding of and enhance our familiarity with this financial instrument.

We start now with a GDP-linked bond. We assume that the sovereign issues a bond of face value equal to 1 with a 20-year horizon. (Many papers have argued that bonds of this type should be perpetual and involve more than one business cycle (see [17]). This is beneficial for both issuers and investors). The bond depends on an index, namely g_t. Thus, the coupon rate δ_t of the bond is a function f of g_t: $\delta_t = f(g_t)$. (We refer to Section 5 for the analysis of several scenarios concerning the function f). Therefore, investors receive a coupon $\delta_t = f(g_t)$ in year t, which is connected to the growth rate of the economy.

It is clear that the feedback the bondholders receive depends on a sequence:

$$\bar{G} = (g_{t_1}, g_{t_2}, \ldots, g_{t_{20}}) \in \mathbb{R}^{20}$$

which we call a *path*. From Section 2, we know that g_t follows a stochastic process. Consequently, the path $\bar{G}$ is also stochastic and cannot be predicted.

We now consider a plain vanilla bond with a constant internal rate of return (IRR) equal to r. Then, the following equation can be established:

$$1 = \frac{f(g_{t_1})}{1+r} + \frac{f(g_{t_2})}{(1+r)^2} + \ldots + \frac{1+f(g_{t_{20}})}{(1+r)^{20}} \qquad (3.1)$$

We fix a path $\bar{G} = (g_{t_1}, g_{t_2}, \ldots, g_{t_{20}}) \in \mathbb{R}^{20}$. Then, the above equation can be solved, and we can obtain the value of IRR r of the vanilla bond for the specific path $\bar{G}$, $r = F(\bar{G}, f)$. The value of r depends on the path $\bar{G}$ as well as on the indexation scenario f that has been chosen for the GDP-linked bond.

However, the paths $\bar{G}$, i.e., years when the economy faces some crisis or has a bump, follow some stochastic process. To overcome this obstacle, we use Monte Carlo simulations in order to run a range of possible paths and, then, to calculate the expected value over all paths. The final result is an equation of the form:

$$r = \mathbb{E}_{\bar{G}}\big(F(\bar{G}, f)\big) = R(f). \qquad (3.2)$$

Equation (3.2) describes the internal rate of return (IRR) of a plain vanilla bond that corresponds to the indexation scenario f of the growth-indexed bond that has been chosen. Since, these GDP-linked bonds take into account the years of recession, when the payments are smaller, and the years of positive growth, when the payments are bigger and compensate for the "bad" periods, the next inequality should hold:

$$\min f \le r \le \max f.$$

4. Data analysis and implementation of the model for the Greek economy

In our research, we gathered historical data for Greece over the past 60 years. Figure 1 shows the real GDP. From this diagram, with the use of the Hodrick-Prescott filter, the logarithm of the GDP series is broken down

into a trend $\ln Y_t$ and a cyclical component $\ln G_t = g_t$ shown in Figures 2 and 3, respectively.

The cyclical part corresponds to a stationary process which, in continuous time, may be modelled by an Ornstein-Uhlenbeck process as described in equation (2.4). The discrete analogue of this process is the autoregression model AR(1) of order 1 and is obtained as follows: we consider dt to be equal to 1 period time, and we replace dg_t with $g_{t+1} - g_t$. Hence, we have:

$$g_{t+1} = g_t - k \cdot g_t + V(W_{t+1} - W_t)$$

FIGURE 1
Gross Domestic Product (GDP) of Greece from 1960-2010

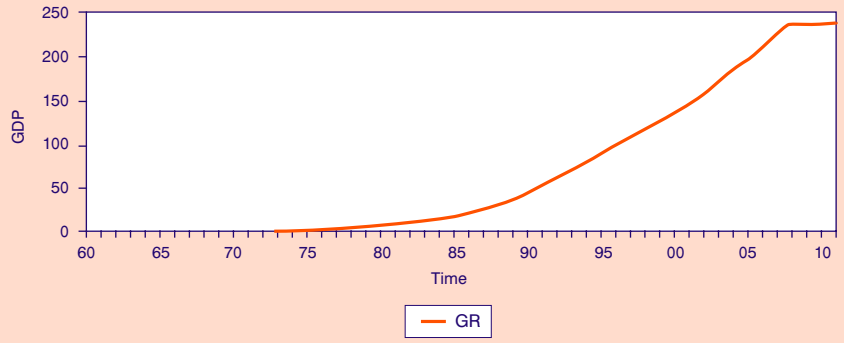


FIGURE 2
The trend of the logarithm of the GDP of Greece

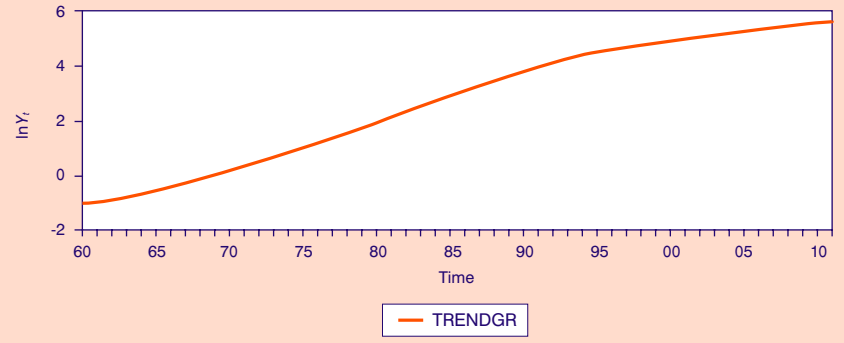


FIGURE 3
The cyclical part gt of the logarithm of GDP (output gap)

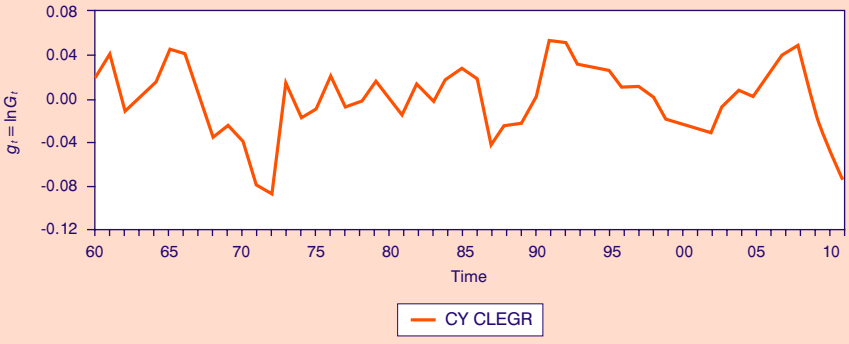
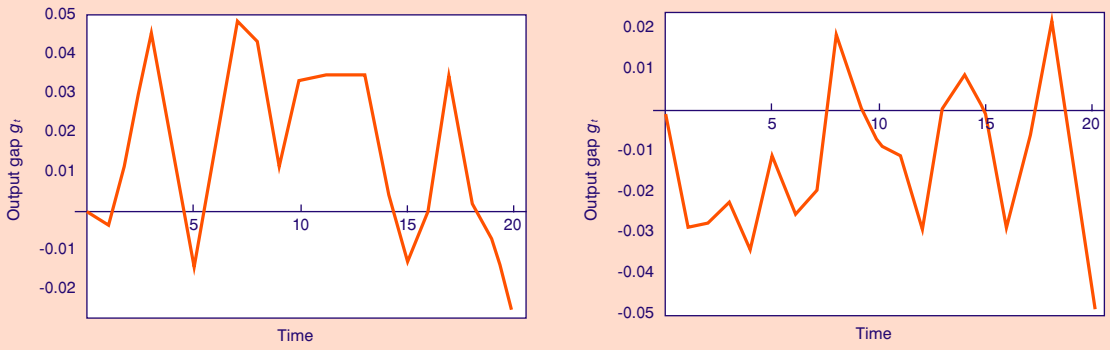


TABLE 1 Autoregression analysis for the output gap g_t

Dependent Variable: CYCLEGR
Method: Least Squares
Date: 06/21/21 Time: 16:44
Sample(adjusted): 1961 2011
Included observations: 51 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
CYCLEGR(-1)	0.597504	0.122319	4.884791	0.0000
R-squared	0.322979	Mean dependent var		-0.000342
Adjusted R-squared	0.322979	S.D. dependent var		0.032744
S.E. of regression	0.026942	Akaike info criterion		-4.370852
Sum squared resid	0.036293	Schwarz criterion		-4.332973
Log likelihood	112.4567	Durbin-Watson stat		1.698354

FIGURE 4
Two scenarios for the output gap g_t in a 20-year horizon



or equivalently,

$$g_{t+1} = (1-k)\cdot g_t + V(W_{t+1} - W_t). \qquad (4.1)$$

The process described in Equation (4.1) was fitted to the macroeconomic data of the Greek economy and the results are depicted in Table 1.

The dependent variable CYCLEGR refers to g_t, and CYCLEGR(–1) indicates that we make an autoregression analysis with g_{t-1}. The result shows that the coefficient, i.e., the coefficient $1-k$ of equation (4.1), equals $0.597504 \approx 0.6$. It follows that $k \approx 0.4 = 40\%$. Furthermore, the standard error (S.E.) of regression (which is equal to 0.026942) is an estimate of the standard deviation of the "true noise" in g_t, i.e., of the term $V(W_{t+1}-W_t)$. It is now known that, if W_t is a Wiener process, then $W_{t+1}-W_t$ follows the normal distribution with mean 0 and variance $t+1-t=1$. Consequently, $V(W_{t+1}-W_t) \sim N(0, V^2)$ and, hence, its standard deviation is equal to V. Hence, $V = 0.026942 \approx 0.027$. Therefore, this analysis suggests that the parameters of the continuous process may be estimated as $k = 40\%$ for the mean reverting coefficient and $V = 2.7\%$ for the volatility of the model.

This process allows the production of numerous fictitious output gap scenarios for a twenty-year horizon. For instance, Figure 4 presents two of these scenarios, which were randomly chosen.

Simulations can be run over a range of these scenarios (for example, 106 such scenarios), where several bond indexations can be considered. Then, one can consider the expected value over all these paths and to decide upon the contract design and the parame-

ters of the GDP-linked bonds. The main pricing scenarios for the GDP-linked bonds are presented in the next section.

5. Bond indexation and scenarios

The pricing and the contract design for GDP-linked bonds are not as clear as the corresponding tasks for the case of vanilla bonds. However, these issues have to be addressed and familiarity will contribute to the acceptance of growth-indexed bonds.

In this section, we revise several scenarios, which have been proposed by the authors cited in the bibliography, for designing the contract of a GDP-linked bond. We also apply them to the case of Greece. We study their equivalence with plain vanilla bonds and analyse the obtained results. As it has been suggested, the pricing of a bond should be as simple as possible to be easily understood.

5.1. Scenario 1: We start with a very simple bond indexation, given by:

$$\delta_t = \begin{cases} 0, & \text{if } g_t < 0 \\ c, & \text{if } g_t \geq 0. \end{cases}$$

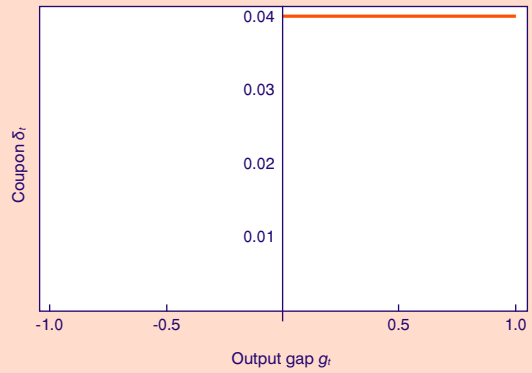
The above formula can be written in a more compact form, as follows: $\delta_t = c \cdot \frac{sign(g_t)+1}{2}$, where for any real number x, $sign(x)$ denotes the sign of x, and we also make the convention $sign(0)=1$ (i.e. $sign(x)=1$ if $x \geq 0$ and $sign(x)=-1$ otherwise). Indeed, if $g_t \geq 0$, then $sign(g_t)=1$, hence $c \cdot \frac{sign(g_t)+1}{2} = c \cdot \frac{1+1}{2} = c$. Similarly, if $g_t<0$, then $sign(g_t)=-1$, hence $c \cdot \frac{sign(g_t)+1}{2} = c \cdot \frac{-1+1}{2} = 0$.

In this scenario, sovereigns have to pay a constant coupon c in year t, if the dynamics in the economy are positive, i.e., when $g_t>0$. This coupon is constant and does not depend on how big the value of g_t is. Otherwise, there is recession ($g_t<0$), then investors receive a zero coupon in year t.

The pay-off function $\delta_t = f(g_t) = c \cdot \frac{sign(g_t)+1}{2}$ is quite simple in this scenario, and its graph is shown in Figure 5.

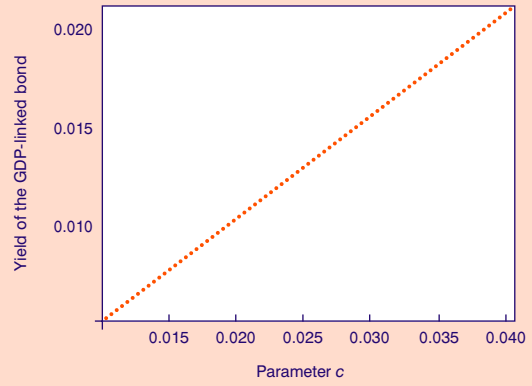
5.1.1. Equivalence with vanilla bonds. In this case, we can repeat the procedure of Section 3 and obtain the correspondence between vanilla bonds and GDP-linked bonds described in that section. More precisely, in this scenario, a path is actually given by a sequence:

FIGURE 5
The graph of the pay-off function $\delta_t = c \cdot \frac{sign(g_t)+1}{2}$, of the first scenario, with $c = 0.04$



Note: The *x*-axis shows g_t, while the *y*-axis shows the coupon rate.

FIGURE 6
Correspondence between a GDP-linked bond, pricing with the first scenario, with the IRR of a plain vanilla bond



Note: The *y*-axis is the required yield of the bond. The *x*-axis represents the parameter *c* of the GDP-linked bond. We observe that investors seeking yield *x*% should invest in a GDP indexed bond with $c=2x$.

$$\bar{G} = (a_1, a_2, \ldots, a_{20}) \in \{0,1\}^{20}.$$

In other words, $a_t=1$ means that the economy is on the rise and investors receive a positive coupon c. On the other hand, $a_t=0$ implies $g_t<0$ and a zero coupon for bondholders. Thus, equation (3.1) takes the form

$$1 = \frac{a_1 \cdot c}{1+r} + \frac{a_2 \cdot c}{(1+r)^2} + \ldots + \frac{1+a_{20} \cdot c}{(1+r)^{20}}.$$

For every path of g_t, the coupon scheme of the GDP-linked bond and the equivalent IRR were calculated. Finally the mean IRR for all scenarios was calculated for a fixed coupon c (of the GDP-linked bond). The relation of the coupon c of the GDP-linked bond and the equivalent rate of an ordinary bond is depicted in Figure 6 above. This implies that for the given historic characteristics of the Greek economy, a rational investor seeking yield x% would be ready to invest in a GDP-linked Greek bond provided the coupon rate for the good periods is of size $2x$%.

The aforementioned remark, a consequence of the fact that the slope of the straight line in Figure 6 is 2, carries important information for the course of the Greek economy in the coming years. In particular, it turns out that the dynamics of the economy is such that the good years to come (positive output gap g_t) are equal to the bad years to come (negative output gap g_t). This amounts to the fact that in the good periods, the investor is going to receive a coupon $2x$% to compensate for the bad periods.

5.2. Scenario 2: We now proceed with an indexation similar to the previous one, but a bit more sophisticated. In this scenario, the bondholder receives at year t a coupon δ_t given by the following expression (see [7]):

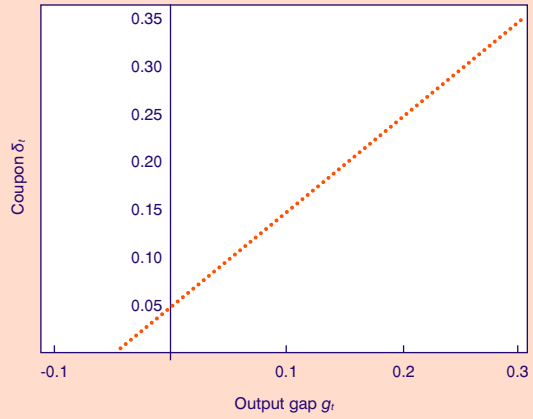
$$\delta_t = \max\{0, \alpha\% + g_t\},$$

where α is a positive number called the *lag* of the bond. This number has to be specified in the contract design process between the issuer and the bondholder.

Therefore, with respect to this scenario, the country issuing the bond pays no coupon if $\alpha + g_t < 0$, i.e., when GDP declines by more than α%. When g_t exceeds the threshold $-\alpha$, then the sovereign has to pay a coupon to its creditors equal to $\alpha + g_t$. The bigger the g_t, the bigger the coupon. We also notice that there is no upper boundary for δ_t, that is, in years when a significant increase in GDP is witnessed, the sovereign has to pay a large coupon. The pay-off function is shown in Figure 7. The lag α of the bond enables the creditor to be paid even in the "bad" periods of the economy; the larger the α, the more "bad" periods the creditor is being paid. Such an option might be more appealing to some creditors as more "risk-free".

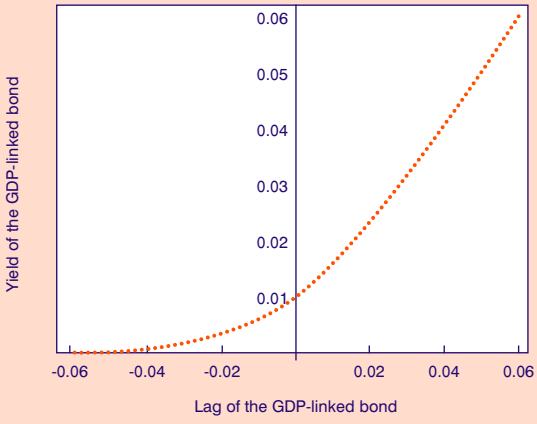
5.2.1. Equivalence with vanilla bonds. Fixing the lag α%, we use simulations over a range of possible scenarios for the growth rate g_t in a 20-year horizon. For every scenario, the equivalent IRR was calculated and then the mean value of IRR was calculated for all possible scenarios. The result is given in Figure 8. The x-axis of the diagram represents the values of α, while the y-axis gives the corresponding yield of the GDP-linked bond.

FIGURE 7
The graph of the pay-off function $\delta_t = \max\{\alpha + g_t, 0\}$, of the second scenario, with $\alpha = 0.05$



Note: The x-axis shows g_t, while the y-axis shows the coupon rate.

FIGURE 8
Correspondence between a GDP-linked bond, pricing with the second scenario, with the IRR of a plain vanilla bond



Note: The y-axis is the required yield of the bond and the x-axis represents the parameter a of the GDP-linked bond.

5.2.2. Analysis and discussion. From this scenario several interesting remarks emerge. First of all, let us consider the case $\alpha = 0$. In this scenario, the pay-off function takes the form $\delta_t = \max\{0, g_t\}$. Therefore, the coupon depends only on the diversion from the trend g_t. If $g_t \geq 0$, investors receive a coupon equal to g_t, whereas if $g_t < 0$, then sovereigns have to pay no coupon.

In Figure 8, we observe that the yield of this GDP-linked bond is relatively small and equal to 1%. This result contains some information about the macroeconomic features of the Greek economy. The conclusion that can be derived is that in a 20-year horizon, the dynamic of the GDP is small on average. Years when g_t is big may be followed by periods where g_t is small and vice versa. Consequently, the yield of the GDP-linked bond is low.

Another observation is that when $a>0.01$, the diagram of Figure 8 becomes linear and the function that is described is close to the line $y = x$. Therefore, if investors seek to achieve a yield equal to 6%, then they should agree on lag $a = 6\%$. That is, the sovereign has to pay no coupon when its GDP declines by 6% or more, whereas it has to pay a coupon 6% + g_t if $g_t>-6\%$. (For instance, if $g_t=0$, then $\delta_t=6\%$, while for $g_t=2$, it follows that $\delta_t=8\%$).

5.3. Scenario 3: In our third scenario, the indexation of the growth-indexed bond is given by the expression:

$$\delta_t = \delta_* + b \cdot \max\{0, g_t + a\}.$$

Compared to the previous pricing scenario, the above formula contains two further characteristics. Firstly, a minimum coupon δ_* (called the *floor* of the bond) has to be paid every year regardless of the economic growth or recession. An extra coupon is paid when the economic growth g_t exceeds some contractually specified threshold $-a$. Secondly, the (also predefined) positive number b has been introduced in the above formula. This parameter multiplies the extra coupon and, thus, it determines the proportion of the growth of real GDP that will be paid to the investors. Bigger values of b induce more sensitivity to the coupon δ_t. We call b the *slope* or *gradient* of the bond.

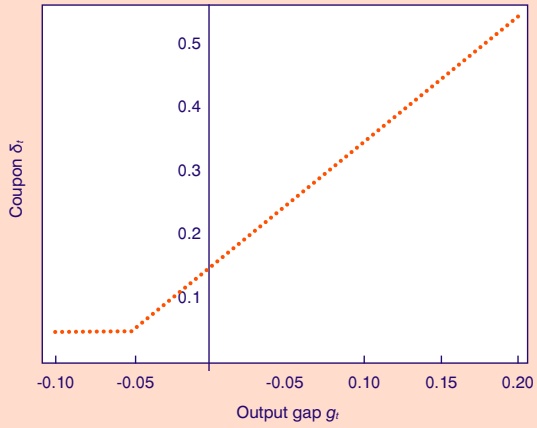
This scenario, which involves a minimum coupon δ_*, clearly reduces insurance for issuers. However, it is more attractive to investors, who may insist on the presence of the floor δ_*. The pay-off function for this scenario is presented in Figure 9.

Finally, we note that the parameters of the bond (i.e., δ_*, b and a) can be seen in the above graph of the function g_t. More precisely, the floor δ_* is equal to the distance of the horizontal line from the x-axis. The lag a is placed at the point where the slope of the graph changes. Finally, the parameter b is equal to the gradient of the non-horizontal line of the graph.

5.3.1. Equivalence with vanilla bonds. Figure 10 shows the equivalence between a GDP-linked bond, which has been designed with the third scenario, and a plain vanilla bond. This figure has also been obtained using Monte Carlo simulations over a range of 20-year paths for g_t and considering the mean value over these paths.

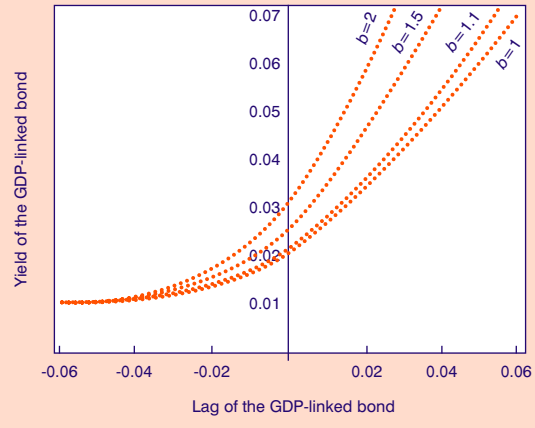
5.3.2. Analysis and discussion. The present scenario seems to be one of the basic tools for our analysis of GDP-linked bonds, since it can reveal several proper-

FIGURE 9
The graph of the pay-off function $\delta_t=\delta_*+b\cdot\max\{a+g_t, 0\}$ of the third scenario



Note: The x-axis shows g_t, while the y-axis shows the coupon rate.

FIGURE 10
Equivalence between a GDP-linked bond, pricing with the third scenario, with the IRR of a plain vanilla bond



Note: The y-axis is the required yield of the bond and the x-axis represents the parameter a of the GDP-linked bond. The different curves correspond to different values of the parameter b (namely, as we move from the left to the right b = 2, 1.5, 1.1, 1).

ties of this instrument. First of all, assume that investors want to achieve a yield equal to 5%. Drawing the horizontal line from the point (0, 0.05) in Figure 10, we see that it meets each diagram at a single point. Every point corresponds to a triple (δ_*, a, b). For instance, in the example under examination, we have the points (0.01, 0.02, 2), (0.01, 0.03, 1.5), (0.01, 0.04, 1.1) and (0.01, 0.043, 1). These triples create different GDP-linked bonds. We now investigate the two extreme points. Of course, several intermediate points can be considered.

- Let us first consider the case $\delta_*=0.01$, $a=0.02$ and $b=2$. Then, the coupon at year t is given by the formula $\delta_t=0.01+2\cdot\max\{0, 0.02+g_t\}$. Therefore, the sovereign pays the floor $\delta_*=0.01$ when its economy declines by 2% or more. Otherwise, it has to pay a coupon given by the previous formula. Consequently, this indexation reduces the pressure on the economy during the "bad" periods. However, because of the slope $b=2$, the coupon increases rapidly in "good" periods. For example, when $g_t=0.02$, then $\delta_t=9\%$. Therefore, when revenues are high, a large part of them should be used to pay off the bond. Hence, they cannot be invested in other activities that would probably contribute to further economic growth.
- The other extreme point in the above diagram corresponds to the triple $\delta_*=0.01$, $a=0.043$ and $b=1$, that is, the coupon is given by $\theta_t=0.01+\max\{0, 0.043+g_t\}$. Therefore, this scenario puts more pressure on the economy during the periods of recession. For example, if $g_t=-3\%$, then the sovereign still has to pay more than the floor $\delta_*=1\%$, namely $\theta_t=2.3\%$, whereas in the previous case, we have $\delta_t=1\%$. On the contrary, during good periods, this bond provides more flexibility to the sovereign. For example, when $g_t=2\%$, the coupon is equal to $\theta_t=7.3\%$, instead of $\delta_t=9\%$ of the previous case.

Summarizing, we can observe that, while the yield of the bond remains the same (equal to 5%), this result can be achieved with several scenarios. Moving from the left curve of Figure 10 towards the right one, we transpose the pressure on the economy from the "good" periods to the "bad" ones and vice versa. Indeed, as we move from the left to the right, b diminishes; therefore, the yield of the bond remains the same as we move from the left to the right only if the coupon pays in deteriorating economic conditions, i.e., only if a increases. We note that large positive values of a imply that the sovereign pays even in "bad" periods of the economy, whereas small negative values of a imply that the sovereign pays in "good" periods of the economy. This explains, in particular, why all the curves in Figure 10 converge for small negative values of a. Consequently, this type of bond gives policy makers the opportunity to choose the best scenario (according to other macroeconomic features of the economy), while the result for the bondholders will remain invariant.

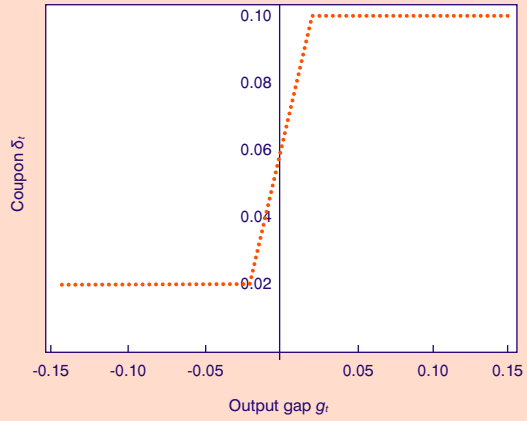
Two final remarks for this scenario: Firstly, we observe that if the lag a is zero, then the yield of the bond has raised from 1% (see Scenario 2) to 2-3%. This increase is due to the minimum coupon $\delta_*=1\%$ and to the slope $b\in[1, 2]$. Secondly, there is no upper boundary for δ_t. Consequently, if the growth rate g_t is very high, the coupon that the sovereign is obliged to pay will be exceptionally high as well. Therefore, the country will have to service the loan, instead of investing its revenues into other profitable activities. This drawback can be solved with the next scenario.

5.4. Scenario 4: In order to address the issue mentioned above, we can design the following coupon:

$$\delta_t=\min\{\delta_*+b\cdot\max\{0, g_t+a\}, \delta_M\}.$$

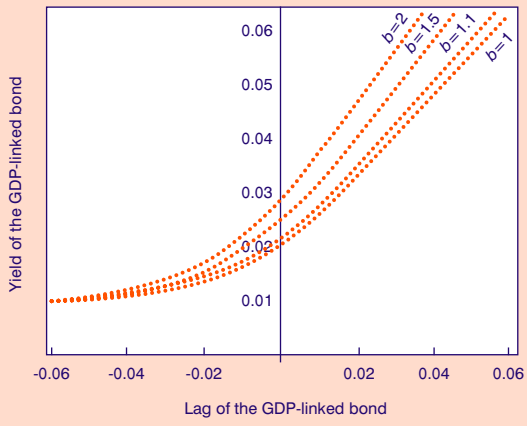
In this case, δ_M is the maximum coupon that the sovereign is willing to offer. We call δ_M the *cap* of the bond. In Figures 11 and 12, we present the graph of the function δ_t and the equivalence of a GDP-linked bond with a plain vanilla bond (obtained with Monte Carlo simulations over possible paths of g_t). Given the curves of

FIGURE 11
The graph of the pay-off function $\delta_t=\min\{\delta_*+b\cdot\max\{a+g_t, 0\}, \delta_M\}$, of the fourth scenario



Note: The *x*-axis shows g_t, while the y-axis shows the coupon rate.

FIGURE 12
Equivalence between a GDP-linked bond, pricing with the fourth scenario, with the IRR of a plain vanilla bond



Note: The *y*-axis is the required yield of the bond and the *x*-axis represents the parameter *a* of the GDP-linked bond. The different curves correspond to different values of the parameter *b* (from left to right: $b = 2$, 1.5, 1.1, 1).

Figure 12, we observe that the analysis of Scenario 3 also applies here.

5.5. Scenario 5: Finally, in the aforementioned scenarios, one can observe that δ_t is linear with respect to g_t. However, this is not obligatory, and one may consider non-linear dependence. For example, one may define (see [5]):

$$\delta_t = \delta_* + b \cdot \sqrt{\max\{0, g_t\}}$$

where $b > 0$. In the equation above, δ_* is the floor, i.e., the minimum coupon payment. An extra coupon has to be paid in year t if the growth rate g_t is positive. The positive constant b determines the elasticity of the coupon to g_t. Because of the square root, a positive value of g_t enforces a bigger increase in the coupon δ_t compared to the previous linear scenarios (recall that the cyclical part g_t is given as a number between 0 and 1).

5.6. Scenario 6: During the decades preceding the economic crisis of 2007-2009, many countries witnessed high growth rates for many consecutive years. In this case, a GDP-linked bond service is more expensive than a plain vanilla bond service. As mentioned in the introduction, many policy makers are not attracted by growth-indexed bonds, because they seem costlier than the usual ones. In this case, the following indexation can be proposed:

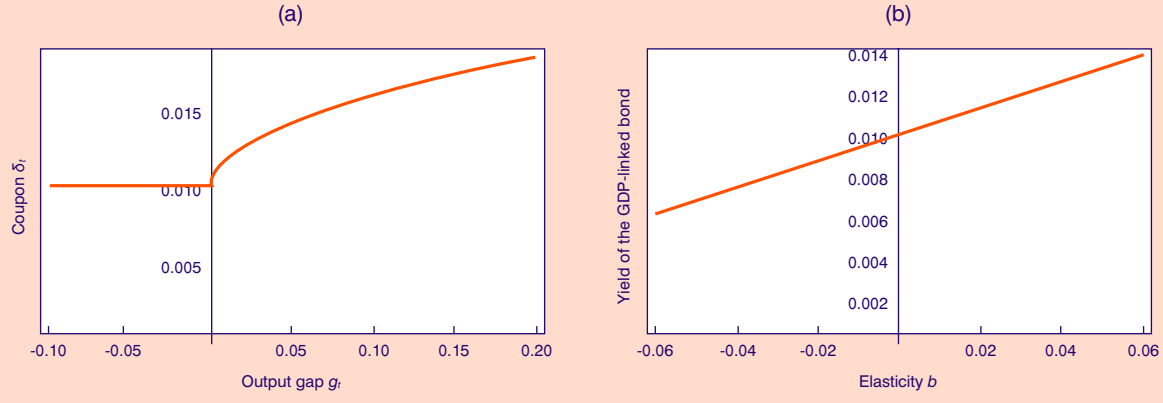
$$\delta_t = \begin{cases} \delta_t^i, & t \in [0, t_0] \\ 0, & t = t_0 - 1, t_0, \ \text{if } g_t > g^* \ \text{for all } t \in [0, t_0 - 2] \end{cases}$$

where t_0 is the horizon of the bond and δ_t^i, $i = 1, 2, 5$ is the bond indexation of any of the previous five scenarios. Therefore, if the economy goes exceptionally well for many years, then the last two years, the investor receives a zero coupon.

6. Contract design and implementation of a GDP-linked bond

The whole contract of a GDP-linked bond must be designed very carefully and in great detail. The imple-

FIGURE 13
(a) The graph of the pay-off function δ_t of the fifth scenario
(b) Equivalence between a GDP-linked bond, pricing with the fifth scenario, with the IRR of a plain vanilla bond



mentation of these bonds to the market place implies that several topics have to be clarified, and a plethora of parameters have to be specified. In this section, we describe two key steps concerning the implementation of a GDP-linked bond to the market.

- *Indexation parameters of the bond*. The first key step of a GDP-linked bond is the selection of the indexation. The analysis of the previous sections reveals that there is a range of possible scenarios with different characteristics. Firstly, the investor decides upon the yield that they must receive, and the pay-off function giving the coupon rate δ_t. Then, some parameters have to be chosen (see, for example, scenarios 3 or 4 of the previous section). Each parameter selection corresponds to a different pay-off method. For example, the investors may prefer to have a small lag and a big slope (which implies that they will receive bigger payments in good periods), or they may prefer a larger lag with smaller slope (which means that they receive some payment even in years of recession, but in good periods their coupons will be smaller). The key characteristic of growth-indexed bonds is that they offer a wide range of choices to investors.

  However, the issuer of the bond may not agree on the design and choices of the investors. Therefore, we have three cases: (a) The country designs the whole coupon and the characteristics of the bond. Then, it presents the product to the market and investors may accept or reject it. (b) Investors design the bond based on the desired yield and their preferences about the payment method. Then, the country has the choice to accept the bond or not. (c) In the majority of cases, however, there is a trade-off between the country and the market and the indexation parameters are determined after negotiations and discussions.

- *Pay-off procedure of the bond*. After the two parties have agreed on the indexation parameters, the contract that is going to be signed must also describe the payment method of the bond. The coupon is not fixed, but depends on the cyclical part g_t of the country's GDP. Therefore, g_t of the year t must be known so that the coupon δ_t can be calculated. To this end, the first step is to determine, in the beginning of the contract, the trend $\bar{Y}_t$ of the country's Gross Domestic Product. The trend should be calculated using the macroeconomic characteristics of the country. The sovereign should present historical data of the economy as well as the method for calculating the trend $\bar{Y}_t$ of the GDP. Then, every year, the cyclical part g_t has to be calculated. The method of calculation should be agreed upon. For instance, it may be conducted either by the official or the private sector. An issue that may occur is that, in any country, there is a delay in the measurement of the real GDP. In some cases, the annual GDP is revised even after some years. However, a big delay in the calculation of the coupon may imply that the sovereign has already entered a period of recession and is unable to pay a big coupon. Therefore, the time when the calculation should take place must also be agreed upon.

- *Other aspects of the bond*. Needless to say, the contract design of a GDP-linked bond is not restricted to the previous two steps. Several other negotiations should take place and several other parameters should be agreed upon. For instance, issues related to the validity of the data should be addressed as well as the legal rights of the investors in the case where the sovereign is unable to fulfil its obligations. For more issues related to the design of a growth-indexed bond, we refer to [17].

7. Conclusions

The 2007-2009 mortgage crisis and the subsequent crisis in the Eurozone put the economies of developing countries under pressure. Furthermore, the recent COVID-19 pandemic reduced unexpectedly the prospects of the global economy. During the next years, other challenges may also occur. For instance, countries all over the world may have to address severe problems due to the climate change.

Many institutions, scientists and economists argue that it is high time we considered new types of bonds, that is, bonds connected with economic growth. It is generally accepted that instruments of this type reduce the probability of default, which is costly for all parties involved (i.e., sovereigns and bondholders). Consequently, they should be studied, and their mechanism should be understood as well and as quickly as possible.

In the present work, we studied exclusively the case of GDP-linked bonds. Several other options can also be considered, for instance inflation-indexed, export or commodity-indexed, wage-indexed bonds, etc. A combination of these products is also possible and may provide the ideal results. However, GDP-linked bonds have the advantage that they are easier to understand. Furthermore, data concerning the national GDP are regularly published and forecasts are available from both the official and the private sectors.

Additionally, in all parts of this article, we have assumed that only the coupons of the bonds are related to the dynamics of the economy and the principal remains fixed. There are also studies where both the principal and the coupon are indexed to GDP. In this scenario, the debt to GDP ratio remains stable. However, many investors seek principal protection and, hence, in many cases only an annual coupon can be connected to the economic situation of the country. Furthermore, we have assumed that the principal will be paid at the end of the bond. Therefore, we did not give some definition of default, and we did not examine the case where the sovereign is unable to pay its obligations. This is a significant part in every contract design of a bond; however, we believe that it does not affect the purposes of the present work.

Our primary purpose was to analyze several indexation scenarios for a GDP-linked bond. We used historical data of the Greek economy; however, analogous processes can be applied to other economies as well (provided that the data are available). Furthermore, we investigated the expected yield of the bond and their equivalence with a plain vanilla bond. This methodology makes this instrument more accessible to the parties involved in the issue of the bond (i.e., issuers as well as investors). In our work, we exploited several indexation scenarios which involve a plethora of parameters. They have floors, slopes, lags and caps. Any change in these parameters imposes changes to the bond. The bond that emerges may transpose the pressure on the economy from the bad periods to the good ones or vice versa. From a wide range of indexation scenarios, we chose six basic methods and analyzed their equivalence with the yield of a plain vanilla bond.

The analysis we performed revealed several interesting properties of this instrument. Firstly, this type of bond can be used as a measure for the macroeconomic behaviour of the country's economy and carries information for its dynamics in the next economic cycle. Secondly, GDP-linked bonds

a) reduce the probability of default, which is a costly scenario for both bondholders and issuers;

b) give the opportunity to sovereigns to choose and organize their own policy. Debt repayments fall in periods of recession and are higher in good periods, while the yield of the bond for the investors remains fixed;

c) protect the country's economy from unexpected phenomena, like the recent pandemic or a natural disaster.

Appendix A. Decomposing into growth and cyclical components

Different methodologies have been proposed over the years (see e.g., Hodrick (2020) and references therein) for decomposing a time series into growth and cyclical components. The one better suited for GDP time series is the well-known Hodrick-Prescott (HP) filter (Hodrick & Prescott (1997)). The HP filter decomposes a time series, y_t, into a *growth* or *trend* component, $\bar{y}_t$, and a *cyclical* component, C_t:

$$y_t = C_t + \bar{y}_t . \tag{A.1}$$

It is understood that in the problem under consideration

$$y_t \equiv \ln Y_t, \quad \bar{y}_t \equiv \ln \bar{Y}_t, \quad C_t \equiv g_t. \tag{A.2}$$

The Hodrick-Prescott (HP) filter (for a series y_t) consists of specifying an adjustment rule whereby the trend component of the series y_t moves continuously and adjusts gradually. Formally, the unobserved trend component $\bar{y}_t$ is extracted by solving the following minimization problem

$$\sum_{t=1}^{N} (y_t - \bar{y}_t)^2 + \lambda \sum_{t=2}^{N} \left((\bar{y}_t - \bar{y}_{t-1}) - (\bar{y}_{t-1} - \bar{y}_{t-2}) \right)^2 \tag{A.3}$$

for $\bar{y}_t$. Thus, the objective is to select the trend component that minimizes the sum of squared deviations from the observed series, subject to the constraint that changes in $\bar{y}_t$ vary gradually over time. The coefficient λ is a positive number that penalizes changes in the trend component. The larger the value of λ, the smoother the resulting trend series. The usual practice is to set λ equal to 100 with annual data series (Agenor (2004), p. 361).

It is convenient to express the objective function (A.3) in matrix form:

$$(y - \bar{y})^T (y - \bar{y}) + \lambda \bar{y}^T \nabla^{2T} \nabla^2 \bar{y} \tag{A.4}$$

where y and $\bar{y}$ are $N \times 1$ vectors of the original data and the trend and ∇^2 denotes the *2nd* difference matrix (Mohr 2005). The solution (Danthine & Girardin 1989) of this optimisation problem follows from the first order conditions in matrix form:

$$\bar{y} = (I + \lambda \nabla^{2T} \nabla^2)^{-1} y, \tag{A.5}$$

$$C = y - \bar{y} . \tag{A.6}$$

Let $y_{[1,t]}$ be a time series $y_1, y_2, \ldots, y_t$ in years 1 to t: Equations (A.2), (A.5) and (A.6) yield

$$\bar{y}_{[1,t]} = (I + \lambda \nabla^{2T} \nabla^2)^{-1} y_{[1,t]}, \tag{A.7}$$

$$g_t = y_t - \bar{y}_t . \tag{A.8}$$

Equation (A.7) is a kind of smoothing operation which eliminates the cyclical component from $y_{[1,t]}$ and gives the trend $\bar{y}_{[1,t]}$. Equation (A.8) is an immediate consequence of equation (2.3) and expresses the difference of the logarithms of the trend of GDP $\bar{y}_t$ and the real GDP y_t.

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