Reliable stochastic design of road network systems

Loukas Dimitriou* and Antony Stathopoulos

Department of Transportation Planning and Engineering,
School of Civil Engineering,
National Technical University of Athens,
5 Iroon Polytechniou, Athens 157 73, Greece
Fax: +30-210-772-2404
E-mail: lucdimit@central.ntua.gr
E-mail: a.stath@transport.ntua.gr
*Corresponding author

Theodore Tsekeris

Centre for Planning and Economic Research,
Amerikis 11, Athens 106 72, Greece
E-mail: tsek@kepe.gr

Abstract: This paper investigates the continuous version of the stochastic Network Design Problem (NDP) with reliability requirements. The problem is considered as a two-stage Stackelberg game with complete information and is formulated as a stochastic bi-level programming problem, which is extended to include reliability as well as physical and budget constraints. The estimation procedure combines the use of Monte Carlo simulation for modelling the stochastic nature of the system variables with a Genetic Algorithm (GA), for treating the complexity of this new formulation. The computational experience obtained from a test road network application demonstrates the ability of the proposed methodology to address the need for incorporating reliability requirements and stochasticity into the various system components in the design process. The results can provide useful insight into the evaluation of alternative reliable network capacity improvement plans under the effect of uncertainty on the demand, supply and route choice process of travellers.

Keywords: network design; systems reliability; transportation systems; stochastic system modelling; Stackelberg game; Monte Carlo simulation; Genetic Algorithms.


Biographical notes: Loukas Dimitriou is Research Associate at the Department of Transportation Planning and Engineering of the National Technical University of Athens (NTUA), Greece. He received a Diploma in Civil Engineering with specialisation in transportation from the NTUA. His research activities include the design and analysis of transportation networks, artificial intelligence and systems reliability engineering.

Antony Stathopoulos is a Professor at the Department of Transportation Planning and Engineering, School of Civil Engineering, National Technical University of Athens (NTUA), Athens, Greece. His research and teaching
activities include transportation systems planning and engineering, traffic network analysis and intelligent transportation systems. He has an extensive involvement in ITS research and an active role in demonstration programmes.

Theodore Tsekeris is a Research Fellow in Transportation Economics at the Centre for Planning and Economic Research (KEPE), Athens, Greece. He received a Bachelors degree in Civil Engineering from the City University, London, Postgraduate diploma and MSc in Transportation from the University of London, UK and a PhD in Transportation from the National Technical University of Athens (NTUA), Greece. His professional and research experience spans over the areas of transportation systems modelling, traffic simulation and control, transportation network management and economics.

1 Introduction

The design of road transportation infrastructure plays a key role in servicing the growing demand for travel in an efficient way. The decision processes involved in providing new capacity in urban areas should ensure both increased mobility unhindered from congestion effects and reliable passenger and freight transportation with a predictable and adequate Level of Service (LoS). The design process should consider such a LoS both in everyday life as well as in the case of unexpected events, like those of physical disasters. Particularly in the latter case, the transportation network performance is of crucial importance for facilitating such civil lifeline operations as emergency aid provision, area-wide evacuation and others (Chang, 2003).

In this context, transportation systems can be regarded as engineering systems composed of real-world components with intrinsically stochastic nature. The stochastic modelling of the components of road networks can help identify situations of design failures, which can potentially cause major malfunctions affecting the economic and social life. Despite of its utmost significance, the modelling of reliability has received a relatively limited attention in the standard design practices of transportation systems, in comparison to the design processes followed in other engineering systems.

An increasing number of studies has stressed the significance of considering reliability as an integral part of the design of real-world engineering systems encompassing various fields of the industry. Such industrial fields refer to computer, information and communication systems (Robinson et al., 2006), power transmission and distribution systems, oil/gas production and manufacturing systems (Rupe and Kuo, 2003), logistics (Sharma et al., 2007) and transportation systems (Ando and Taniguchi, 2006). These studies have shown the beneficial implications of the reliable design for enhancing the security, safety and operational performance of the system, customers’ satisfaction and profitability of firms.

In transportation networks, reliability reflects the ability of the system to respond to the various states of the system variables. Several ways have been studied in the existing literature for quantifying transportation network reliability (see Section 2). These studies are typically restricted to consider the reliability of a single component of the transportation system, such as that of capacity or travel time. This present study follows a holistic approach by using the concept of Total Travel Time (TTT) reliability and examining the impact of fluctuations in link capacities and, hence, travel times.
on the design process under conditions of demand uncertainty, in a unified modelling framework. This framework addresses the problem of the continuous version of the Network Design Problem (NDP) with travel time reliability requirements.

Section 2 describes the transportation network as a stochastic system with random components and presents a simulation framework for modelling network reliability. Section 3 considers the Continuous Network Design Problem (C-NDP) as a two-stage Stackelberg game with complete information and formulates it as a stochastic bi-level programming problem, which is extended to include reliability as well as physical and budget constraints. Section 4 describes a solution procedure for the reliable C-NDP with the use of a Genetic Algorithm (GA). Section 5 presents the computational experience gained from applying the proposed methodology onto a test network and Section 6 concludes.

2 Stochastic modelling of network reliability components

A number of different definitions of the reliability of transportation networks has been hitherto introduced and employed in the current literature. These definitions refer to various aspects of the transportation system reliability, such as travel time reliability, connectivity reliability, network flexibility, network variability and others (see Bell and Iida, 1997; Morlok and Chang, 2004; Stathopoulos and Tsekeris, 2006). This present study investigates the reliable road network design, in terms of ensuring the system operation within a prespecified LoS. The current approach is based on the reliable design of a stochastic system. The measure of the system reliability is expressed with the TTT, since such a metric can depict the stability of the system (Bell and Iida, 1997). The TTT reliability is defined as the probability that network TTT will be less than some predefined bound. The stability of the transportation system lies on the intrinsic uncertainty pertaining to the system operation.

By and large, the measure of uncertainty in the various engineering systems is associated with the stochastic nature of the system components and the models employed to replicate real-world operations (Meredith et al., 1985). Several methods have been proposed in the literature for the stochastic design of systems with reliability requirements. Such methods typically seek to solve the reliability-maximising resource allocation problem in unreliable network systems, through maximising the probability that resources (flows) can be transmitted successfully from origin (source) nodes to destination (sink) nodes.

Recently, Lin and Yuan (2003) addressed the flow reliability problem, in terms of the probability that flow satisfies the demands simultaneously for multiple node pairs of a directed capacitated-flow network, through calculating a family of lower boundary points for servicing such demands in terms of minimal paths. Also, Yeh (2004) evaluated the multistate network reliability as the probability that flow satisfies the demands such that the total budget (maintenance cost) constraints are satisfied. Moreover, the resource allocation problem has been integrated with the reliability evaluation process, firstly, assuming that only the characteristics (arc capacities) of the flow network change and considering the resource demand as fixed (Hsieh and Chen, 2005; Hsieh and Lin, 2003), and, secondly, assuming that either the characteristics of the flow network or the resource demand change (Hsieh and Lin, 2006).
In the field of transportation and logistics, Jain et al. (2006) suggested a novel High Intelligent Time (HIT) Petri-net for modelling the uncertainty of information flows and complex interactions between various system components in the intermodal transportation chain, in order to minimise the total transportation time of the load along the whole chain. Also, Faisal et al. (2007) proposed a Risk Mitigation Environment (RME) for modelling different variables associated with the risk environment in the supply chain process, along with their interdependencies, by using graph theory and matrix methods. Such an approach focuses on managing the risks involved in the operation of the supply chain systems. Sharma et al. (2007) examined the problem of resource allocation and failure behaviour of an industrial system (paper machine) through employing a risk-ranking approach based on a fuzzy decision-making system. Their approach allows incorporating subjective knowledge in the Failure Mode and Effect Analysis (FMEA) so that understand and predict the uncertain behaviour of the system in a more flexible manner, and it helps select suitable maintenance strategies to improve system performance.

In transportation networks, the uncertainty can be considered with regard to three main components: the travel demand, the supply which, in turn, affects travel times and the users’ characteristics. The travel demand in urban transportation networks can be considered as recurrent, under the typical operating conditions (Stathopoulos and Karlaftis, 2001). Nonetheless, demand patterns may experience several disturbances during the operational life of a network. These disturbances can be caused by spatial and temporal (random or non-random) variations of trip flows between Origin-Destination (O-D) pairs, special events in certain network localities, link closures or failures and the nature of the day-to-day route choice process of users. This study simulates the demand between each O-D pair as a Gaussian distributed random variable.

The variations of supply (capacity) can be considered as a common phenomenon in transportation networks. There are several factors influencing capacity, including the composition of traffic, congestion effects, road works and random phenomena like incidents. Since the link and path travel times are closely related to link capacities, the travel time reliability depends on the fluctuations of link capacities, which are usually referred to as link capacity degradation. The problem of considering network reliability in terms of the link capacity degradation has been extensively investigated in the literature (Chen et al., 2002; Cho, 2002; Du and Nicholson, 1997; Yang and Bell, 1998). These studies have shown that fluctuations in link capacities lead to fluctuations in total network capacity and, subsequently, decrease TTT reliability.

In this present study, the capacity of each network link is treated as a Gaussian random variable in order to model the stochastic nature of the link travel times related to capacity fluctuations. This approach is preferable since it allows link travel time variability (which is the result) to be modelled with respect to its causal phenomenon (which is the link capacity variability), in comparison to standard approaches which describe travel time by a normal distribution (Bell and Iida, 1997).

The route choice process has been widely recognised to have a stochastic nature among the network users, particularly in terms of their perception of travel time between alternative routes. The current study adopts a stochastic route choice assumption in order to model users’ preferences. Such an assumption leads to the Stochastic User Equilibrium (SUE) conditions, which involves a stochastic demand assignment procedure (see Section 3).
The Monte Carlo simulation is a widely used and well-documented framework for supporting the design of real-world engineering systems with stochastic components and the evaluation of the system reliability. This simulation framework is employed here as a common basis for modelling the stochastic behaviour of network reliability components, including the demand, supply and route choice process.

More specifically, the Monte Carlo simulation, at each epoch, represents the network state by changing the values of demand between each O-D pair and link capacities, based on the distributions previously defined, and, then, estimating the route choice response of users to link degradation and the properties of TTT. This response is obtained through the stochastic UE assignment of demand onto the paths and links of the network. The simulation of the pattern of both demand and supply components reflects all possible states of the system attributes that can be observed along the duration of its operational life.

This present simulation procedure employs random numbers from normal distributions in order to represent the variations in demand and capacity. The standard procedure of Box and Muller (1958) is applied here for the random number generation from a normal distribution, as follows:

\[ m = \mu + \sigma n \]  
(1)

where

\[ n = \sqrt{-2 \ln (R_1)} \cos (2\pi R_2) \]  
(2)

The disturbance \( n \) is a standard Gaussian process \( N(0,1) \), which allows the production of random numbers \( m \) from \( N(\mu, \sigma) \), with \( R_1 \) and \( R_2 \) be random numbers from the uniform \((0, 1)\) distribution. At each iteration of the Monte Carlo simulation, the UE assignment model estimates a new link traffic pattern among the alternative network routes, together with the values of link and path travel times, and TTT. Hence, at the end of the Monte Carlo simulation, the appropriate information concerning the distribution of the TTT is supplied to the design process. Despite that link capacities are assumed here to follow a normal distribution and are uncorrelated with each other, alternative formulations could be used in the current solution framework for considering situations where link capacities are correlated and follow non-normal distributions, based on procedures estimating correlated random numbers.

3 Formulation of the reliable stochastic system design problem

The objective of the NDP concerns the selection of those network improvements (link additions, link capacity improvements) which can result in maximum benefit, subject to physical and budgetary constraints. As in many other transportation planning problems, the optimum network design is affected by decisions made on multiple hierarchical levels, which concern both the demand and supply properties of the system (Fisk, 1984, 1986; Zhang et al., 2005). Hence, the NDP can be modelled and analysed as a two-stage leader-follower Stackelberg game (Von Stackelberg, 1952) with perfect information. In such a game-theoretic situation, optimal solutions are enquired by taking into account the interaction between the two-stage alternative players and physical and
budgetary constraints, while the demand and supply attributes of the system are considered as known. In the first stage, improvements are imposed to the network configuration by an authority (leader), who seeks to a system optimum solution in accordance with the physical and budgetary restrictions. In the second stage, users (followers) respond to the authority’s improvements by making decisions collectively, in order to optimise their own performance measures, which are typically based on some inverse demand cost function.

There are two distinct types of NDP, namely, the C-NDP and the Discrete Network Design Problem (D-NDP). The former type implies improvements on the capacity of the network links, which are expressed as continuous variables (i.e. number of vehicles), while the latter type implies additions on the network, which are expressed as discrete variables (i.e. number of lanes, link additions). This present study uses the C-NDP paradigm, namely, it applies the reliable stochastic system design approach into the problem of determining the optimal network capacity improvements in the form of continuous variables.

A large number of alternative approaches for formulating the NDP can be found in the existing literature. Most of these approaches recognise the NDP as a Stackelberg game and formulate it as a bi-level mathematical programming problem. In this bi-level problem, the upper-level problem estimates optimum enhancements in link capacities, while the lower-level problem provides the reaction of users to the changes made in the upper-level problem. Exact solutions of the NDP can be found only in the case where both upper- and lower-level problems are expressed as linear functions (Le Blanc and Boyce, 1986). In different case, both types of the NDP form a non-linear, non-convex optimisation problem, since the set of their constraints is composed of non-linear functions. In particular, the lower-level problem, which imposes constraints to the upper-level problem, is a non-linear non-convex problem, although it has a unique solution. Although the NDP can be also formulated as a single-level optimisation problem, putting the lower-level problem into the set of the upper-level problem constraints, for example, by using the Variational Inequality (VI) formulations (Friesz et al., 1992) or expressing the NDP as a Mathematical Program with Equilibrium Constraints (MPEC) (Lim, 2002), does not eliminate the issue of non-convexity. A comprehensive review of the approaches used for formulating and solving the NDP can be found in Friesz (1985). The standard bi-level programming formulation of the C-NDP is extended here to take into account both the stochasticity of the system variables and the reliability requirements.

Consider a network composed of \( L \) links, \( R \) origins and \( S \) destinations. The travel demand \( q_{rs} \) gives rise to equilibrium flows \( f_k^{rs} \) along path \( k \in K_{rs} \) connecting \( r-s \) pair and to equilibrium flows \( x_a(y) \) along link \( a \), with \( \delta_{ka}^k \) be the path-link incidence variable and \( c_{rk} \in C^{rs} \) be the cost of travelling along the \( k \)th path between \( r-s \) pair. The travel cost, at equilibrium state, of some link \( a \) with capacity \( y_a \) is denoted as \( c_a(x_a(y)) \), with \( y \) be the total network link capacity. Also, \( V_a(y_a) \) denotes the monetary expenditures for improving the capacity of link \( a \), \( B \) is the total available construction budget for network capacity improvements and \( \theta \) is a factor converting monetary values to travel times. Then, the upper- and lower-level problem, which
Reliable stochastic design of road network systems provides the UE path and link flows, of the stochastic continuous equilibrium NDP can be given as follows:

**Upper-level problem:**

\[
\min_{y} F(x, y) = \sum_{a \in A} \left( E\left[ c_a\left(x_a(y), y_a\right)\right] + \theta V_a(y_a) \right), \quad \forall a \in A
\]  
\[\text{s.t.} \quad 0 \leq y_a \leq u_a, \quad \forall a \in A \tag{3} \]
\[
\sum_{a \in A} V_a(y_a) \leq B, \quad \forall a \in A \tag{4} \]
\[
P\left( \sum_{a \in A} c_a\left(x_a(y), y_a\right) x_a(y) \right) \leq T, \quad \forall a \in A \tag{5} \]

**Lower-level problem:**

\[
\min_{s} G(x) = -\sum_{s} q_{rs} E\left[ \min_{s} \left( c_s^{k}\left(y_s\right)\right) \right] + \sum_{a} x_a c_a\left(x_a\right) - \sum_{a} \int_{0}^{\infty} c_a(\omega)d\omega, \quad \forall k, r, s \tag{7} \]
\[\text{s.t.} \quad f_{rs}^{k} = q_{rs}, \quad \forall k, r, s \tag{8} \]
\[
x_a = \sum_{s} f_{as}^{k} \delta_{as}^{k}, \quad \forall a \in A \tag{9} \]
\[
f_{rs}^{k}, x_a \geq 0, \quad \forall a \in A \tag{10} \]

In the upper-level problem, \( F(x, y) \) represents the objective function of the C-NDP, wherein the first component refers to the travel cost expressed in terms of the expectation \( E \) of network TTT, and the second component corresponds to the total expenditures (in time units) for capacity improvements. Inequality (4) represents the physical constraints for capacity improvements at link \( a \) through defining an upper limit \( u_a \), while inequality (5) imposes the budgetary constraints. The reliability requirements are introduced here through the risk assessment of the TTT, in the form of an additional constraint, that is, inequality (6). The network reliability metric is expressed by the measure

\[
P\left( \sum_{a \in A} c_a\left(x_a(y), y_a\right) x_a(y) \right) \leq T \]

which represents the probability of the TTT to be lower than or equal to a prespecified upper limit \( T \), with \( Z \) defining the acceptable confidence interval ( \( 0 \leq Z \leq 1 \) ) for this hypothesis.

The lower-level problem refers to the demand assignment process, which is described by Equations (7)–(10) and provides the reaction of users to the network capacity changes made at the upper-level problem, based on the expected (perceived) value \( E \) of the path.
travel cost $c_{rs}^{k}$. Specifically, the SUE conditions are obtained by adopting a logit-based formulation for modelling the route choice behaviour, through determining the probability $P_{rs}^{k}$ that a traveller chooses to use path $k$ between $r - s$ pair (Sheffi, 1985).

According to the assignment logic employed here, equilibrium flows $f_{rs}^{k}$ are estimated by iteratively assigning the demand onto the network paths and links, until reaching the SUE conditions. In the current study, the Method of Successive Averages (MSA) is used to estimate the stochastic equilibrium link flows.

In contrast with the standard formulation of the C-NDP, where the lower-level problem is solved once (single pass) for each construction plan imposed from the upper-level problem, the proposed formulation involves an iterative procedure for estimating the users’ responses and the properties of the TTT distribution. In particular, the current solution procedure, which is described in the following section, is carried out by iteratively performing simulation of the stochastic system variables and checking for the network reliability requirements through inequality (6).

4 Estimation method for the reliable stochastic C-NDP

Several approaches have been presented in the literature for solving the C-NDP, which can be classified into:

1. gradient-based methods (Chiou, 2005)
2. gradient-free meta-heuristic methods, including the Hooke and Jeeves technique (Abdulaal and LeBlanc, 1979), equilibrium decomposed optimisation procedure (Suwansirikul et al., 1987), simulated annealing (Friesz et al., 1992) and GAs (Cree et al., 1998).

All these approaches employ static assignment procedures for calculating the traffic flows, after making the network improvements. Furthermore, models for traffic flow estimation at dynamic disequilibrium states (Friesz and Shah, 2001) and dynamic equilibrium algorithms (Karooonsoontawong and Waller, 2006) have been proposed and tested. A few models have also been considered for treating the uncertainty in the various system attributes, such as in demand (Waller and Ziliaskopoulos, 2001) and travel time (Yin and Iida, 2002). In addition, the C-NDP with travel time reliability requirements has been studied by the use of approximation methods in order to estimate the statistical properties of the system attributes and, particularly, the Probability Density Function (PDF) of the TTT, which ends up with solving a sequential quadratic programming problem (Sumalee et al., 2005). A preliminary investigation of applying a stochastic system design approach for solving the reliable D-NDP was made in Dimitriou et al. (2007).

This present formulation (see Section 3) adds to the complexity of the C-NDP, due to the treatment of system attributes as stochastic variables and the simulation-based risk assessment of TTT, which is incorporated in the set of constraints. Such a formulation results in a highly non-linear and complex problem, whose solution cannot be obtained by using classic derivative-based optimisation procedures. Meta-heuristic stochastic global optimisation methods, such as GAs which are employed in this study, have been proven to be able to tackle problems of increased complexity. GAs, initially introduced by Holland (1975), constitute meta-heuristic population based, derivative-free
optimisation techniques, which exploit the mechanics of natural evolution in order to gradually approach optimality conditions (Goldberg, 1989). They are appropriate for solving non-convex problems, such as this present one, due to their ability to handle variables of stochastic nature and multiple constraints in a convenient way, requiring only information about the performance of a ‘fitness’ function of various candidate states of the system. GAs have been extensively used for solving complex optimisation problems and, particularly, bi-level programming problems in the field of transportation (see Yin, 2000) as well as in many other application fields (for a general review, see Colson et al., 2005; Oduguna and Roy, 2002).

Since the structure of alternative GAs can be found in various specific textbooks on the field of evolutionary computing (e.g. see Goldberg, 1989), this current study provides a concise presentation of the most important elements of the algorithm used here. GAs utilise an initial population of typically randomly chosen feasible solutions combined with a pattern of evolution, forming a mechanism of recursive production of new improved solution populations. The pattern of evolution is based on the principle of natural evolution. According to it, solutions with increased performance, in terms of the values of the objective function, which are likely to lie closer to optimality, have increased probability to be selected, in order to provide the new improved solution population. Each ‘individual’ of the population is a coded set of the problem variables and forms a string of the variable values, referred to as chromosome. In this current study, the values of the variables follow the binary arithmetic coding scheme, that is, each chromosome is a string of ‘0s’ and ‘1s’, which are called allelic values. The mechanism of the population evolution is based on three genetic operations, that is, reproduction, crossover and mutation:

**Reproduction:** this operator performs the reproduction of an intermediate population, referred to as parent population, from which a new, genetically improved population will be produced. There are various methods for the reproduction of the individuals of the parent population, such as the roulette wheel and tournament selection. In this study, the tournament selection method is adopted. In this method, the parent population is formed by choosing the most ‘powerful’ amongst a number of randomly selected individuals from the old population.

**Crossover:** after selecting the parent population, the exchange of genetic material (crossover) among the member individuals is the mechanism that leads to the production of a new improved population. The crossover is performed by randomly mating the individuals and exchanging parts of their chromosomes, according to a prespecified rate (probability of two mates to crossover) and pattern. The current application uses a scattered crossover pattern, wherein randomly selected parts of each chromosome are exchanged to allow the transmission of genetic information among ‘individuals’.

**Mutation:** this operation provides a mechanism for preventing local convergence through randomly altering some allelic values according to a prespecified (typically small, such as <5%) rate.

This current iterative procedure for solving the C-NDP combines the use of the mechanics of natural evolution, as it is based on a GA, with Monte Carlo simulation for modelling the stochastic variables and performing the risk assessment in relation to the reliability requirements. The individuals of the GA population contain candidate
solutions, that is, the values of the capacity improvements that should be made to the network links. For each individual of the population, a Monte Carlo simulation is performed that alters the demand and link capacities, in order to estimate the parameters of the TTT distribution and, hence, the TTT reliability. The steps of this solution procedure are presented in Table 1.

It should be noted that, although the extended use of meta-heuristic techniques, such as GAs, for solving complex problems, the solutions provided by them have met some skepticism in the literature. In particular, Huang and Bell (1998) suggested that alternative heuristic methods can provide very different solutions, since they are heavily dependent on initial conditions and the random search processes incorporated into these methods and the possibility that multiple (local) optimum solutions may exist. In order to address this problem, multiple runs of the GA are performed in the present experimental set-up (see Section 5) in order to confirm that the resulting outcome is the optimal one (or adequately near-optimal).

Table 1 The steps of the procedure used for solving the C-NDP

<table>
<thead>
<tr>
<th>Step 1 (Initialisation)</th>
<th>Production of an initial random population of candidate feasible solutions (link capacity improvements) and selection of the properties of the genetic operators.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Do until convergence:</td>
<td></td>
</tr>
<tr>
<td>Step 2 (Simulation)</td>
<td>Estimation of the TTT reliability for each candidate solution with Monte Carlo simulation.</td>
</tr>
<tr>
<td>Step 3 (Genetic evolution process)</td>
<td>Step 3.1 Check of the consistency of constraints and estimation of the ‘fitness function’ of each candidate solution.</td>
</tr>
<tr>
<td></td>
<td>Step 3.2 Stochastic selection of the ‘fittest’ solution set and performance of the crossover operation among the selected individuals.</td>
</tr>
<tr>
<td></td>
<td>Step 3.3 Mutation of the individuals.</td>
</tr>
<tr>
<td></td>
<td>Step 3.4 Production of a new population of genetically improved candidate solutions.</td>
</tr>
</tbody>
</table>

5 Numerical example and results

The proposed methodology is implemented into a test network in order to get insight into the differences among the alternative construction plans when reliability requirements are added to the C-NDP. The test network is composed of 12 nodes and 23 links, which form 25 paths servicing a single O-D pair (see Figure 1). The total demand \( q \) between the O-D pair is set equal to 80 vehicles per hour (veh/hr). The variance of the normal distribution of travel demand is set equal to the 10% of its mean (set) value. The initial mean capacity \( y_c \) of each link is set equal to 20 veh/hr. The upper allowable bound \( u_c \) for capacity improvement in each link is set equal to 30 veh/hr. The link capacities have been treated as normally distributed random variables, with mean value equal to the theoretical mean capacity and standard deviation that amounts to the 20% of the theoretical capacity.
In order to model travel time variability with respect to link degradation, the standard formulation of the Bureau of Public Road (BPR) has been adopted here for estimating travel time $t_a$ along some link $a$ with free flow travel time $t'_a$, by using the following formula:

$$t_a = t'_a \left(1 + \beta \left(\frac{x}{y_a}\right)^m\right)$$

(11)

The scale parameters $\beta$ and $m$ depend on the network operational characteristics and they have been set here equal to $\beta = 0.15$ and $m = 4$. It is noted that although this study estimates link travel times based on the BPR formula, other formulations could also be employed to take into account congestion phenomena like queues or bottlenecks at network links. The initial (free flow) travel time, which is proportional to the link length, is set equal to 1 min for links 1–17 and 1.4 min for links 18–23. This study uses a conversion factor $\theta$ from monetary units to travel time units equal to unity ($\theta = 1$) and a total available budget $B$ for network construction plans equal to 350 monetary units, which corresponds to the half of the amount needed for expanding the capacity of all links to the upper allowable bound. The stochastic UE traffic conditions are reached after performing 200 iterations based on MSA, which were found to be adequate for providing a stable solution of path flows.

An investigation of the pattern of TTT with respect to the link capacity variability is first undertaken. The results of this investigation indicate that when link capacities follow a normal distribution, then, the distribution of link travel times (Figure 2) and TTT (Figure 3) can be better represented by a positively skewed distribution, like those of the family of lognormal distributions, as opposed to a normal distribution. The solid lines in Figures 2 and 3 indicate the fitted lognormal distribution of link travel times and TTT, respectively over their simulated values for comparative purposes. This outcome is consistent with the physical meaning of the system variables, whose values may increase asymptotically, but they cannot fall below zero.
Figure 2  Distribution of link travel times (in min)
Figure 2  Distribution of link travel times (in min) (continued)
Figure 2  Distribution of link travel times (in min) (continued)
Figure 2  Distribution of link travel times (in min) (continued)

![Link #10](image)

![Link #11](image)

![Link #12](image)
Figure 2  Distribution of link travel times (in min) (continued)
Figure 2  Distribution of link travel times (in min) (continued)
Figure 2  Distribution of link travel times (in min) (continued)
Due to its properties, especially those related to non-negativity and positive skewness, the lognormal distribution can be considered as highly suitable for describing the attributes of route travel time as well as TTT in road network systems. Also, the results demonstrate that the lognormal distribution can be well used to model transportation network reliability, as it extensively occurs in modelling reliability in many other engineering systems. Moreover, the distribution of the TTT is analysed with respect to different levels of link capacity variation. Figure 3 shows that the TTT is significantly influenced by the size of link capacity variability. In particular, when the dispersion of link capacity increases (in terms of the variance) from 10% to 25%, the dispersion of the TTT increases substantially, dropping the reliability of the whole system.

Before applying the proposed method onto the test network, a sensitivity analysis is performed in order to trace those links which mostly influence the TTT. The sensitivity analysis concerns the examination of the effect that one standard deviation increment of
the capacity of each link can have on the standard deviation of TTT. Figure 4 presents the ranking of the links with the largest impact on TTT, in terms of the correlation coefficient between the increment of link capacity by one standard deviation and the standard deviation of the TTT. As it may be expected, these links, which can be considered as the most critical components of the system, are, in order of significance, #17, #1, #9, #4, #14, #18 and #13, since they are servicing the largest portions of demand between the O-D pair.

Moving to the production of solutions for the current C-NDP, a suitable setup of the GA is made. For the given network, the GA includes a population of 50 binary coded ‘individuals’ (problem solution sets), which corresponds to almost the double of the problem size, that is, 23 network link capacities. The crossover and mutation rate are chosen to be 70% and 1%, respectively. A convergence criterion of no significant improvement to the fitness value (<0.1) after 50 generations is preferred, resulting in the convergence to be occurred typically after 100 generations. Finally, for each individual, 200 iterations of the Monte Carlo simulation are performed to obtain the stochastic properties of the system variables.

Figure 3 Lognormal distribution fit of the TTT with different link capacity variations: (a) link capacity variance equals 10% of the mean capacity; (b) link capacity variance equals 15% of the mean capacity; (c) link capacity variance equals 20% of the mean capacity and (d) link capacity variance equals 25% of the mean capacity

An initial solution is first obtained by solving the C-NDP without reliability requirements. The assignment of travel demand onto the initial network (without link improvements) results in a TTT equal to 549 veh-min. The initial solution (see Figure 5(a)) refers to a construction plan composed of improvements on links #17,
#1, #9 and #4, which result in the reduction of the expected TTT from 549 veh-min to 402 veh-min (≈27% reduction). The construction plan obtained from this solution essentially corresponds to the enhancement of those 4 links which are the most influential to the value of TTT, based on the results of the sensitivity analysis shown above (Figure 5). An amount of 40.8 monetary units has been used as construction cost (corresponding to 40.8 veh-min, since $\theta=1$), which, in turn, leads to a total cost (TTT + construction cost) equal to 442.8 veh-min.

This specific construction plan can be considered as defining a threshold beyond which capacity improvements are regarded as too expensive, with respect to the increase of the total cost, in comparison to their contribution to the reduction of the expected TTT, although the total available construction budget is much higher than the construction cost obtained from the solution. In terms of the composite objective function, after $F$ value exceeds that specific point (threshold), additional investments for link capacity improvements increase the component of the construction cost proportionally more than the reduction caused to the TTT component, independently from the link wherein the investments are made.

**Figure 4** Results of the sensitivity analysis: the correlation coefficients between one standard deviation increment of each link capacity to the standard deviation of the TTT.
In addition, a reliability assessment is carried out for the construction plan obtained from the initial solution, in order to estimate the distribution of TTT (see Figure 5(b)). The results of the Monte Carlo simulations demonstrate that the probability of TTT exceeding the upper limit $T = 450$ veh-min is 15% (an amount of 450 veh-min is selected since it represents a 10% increment of the TTT). Next, a new construction plan is formed by adding a reliability requirement, that is, the probability of the TTT to exceed 450 veh-min to be less than 5%. Figure 5(c) presents the new construction plan and Figure 5(d) illustrates the estimated distribution of TTT, as they were resulted from applying the solution procedure of Table 1. As it was expected, more capacity is required in order to increase network reliability. This outcome is consistent with the concept of sparse network capacity, or that of increased system redundancy, in systems engineering terms. In particular, the total construction cost is increased to 67.73 monetary units, while the expected TTT is reduced to 387 veh-min, leading to a total cost of 454.73 veh-min. Table 2 provides a comparison between the results obtained from applying the algorithm into the two different problem settings (without and with reliability requirements).

Table 2 Results of the test network improvements without and with reliability constraints

<table>
<thead>
<tr>
<th></th>
<th>Without reliability requirements</th>
<th>With reliability requirements</th>
<th>% change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total construction cost (veh-min)</td>
<td>40.8</td>
<td>67.7</td>
<td>+60.2</td>
</tr>
<tr>
<td>TTT (veh-min)</td>
<td>402.0</td>
<td>387.0</td>
<td>−3.7</td>
</tr>
<tr>
<td>Total Cost (veh-min)</td>
<td>442.8</td>
<td>454.7</td>
<td>+2.6</td>
</tr>
<tr>
<td>$P(TTT \leq 450$ veh-min)</td>
<td>85.6%</td>
<td>95.8%</td>
<td>+10.7</td>
</tr>
</tbody>
</table>
6 Conclusions

This paper investigated the C-NDP with reliability requirements, by treating the design of a road transportation network as a problem of designing a reliable stochastic system. The current approach is based on the intrinsic modelling of the stochastic properties of the system components, including the demand, supply and route choice process. The C-NDP is formulated as a stochastic bi-level programming problem, which seeks to determine the optimal network capacity improvements, subject to reliability as well as physical and budgetary constraints. The reliability requirements refer to the TTT and they are introduced into the set of problem constraints through defining a threshold for the probability of TTT to be less than a prespecified value.

This present model involves an iterative solution procedure, which combines the use of Monte Carlo simulation for representing (simulating) the system variables and, hence, estimating the travel time reliability, with a stochastic global optimisation method, that is, a GA, for addressing the complexity and non-convexity of the problem. The test implementation showed that the proposed procedure is able to find optimal solutions within the prespecified conditions related to the reliability and the other problem requirements. In particular, the benefits from adopting the new formulation of the C-NDP reflect on the reduction of the TTT, while satisfying the desired level of network reliability and the budget constraints. The results verify that more capacity is needed in order to increase network travel time reliability.

In addition, the results provide useful insights into the stochastic properties of the various components involved in designing a reliable transportation network. Specifically, it is demonstrated that, in consistency with other engineering systems, the lognormal distribution can be well used to model transportation network reliability, provided that the link capacities and travel demand follow a normal distribution. The current methodological framework helps address a number of fundamental, inherent uncertainties associated with the design of transportation networks. Nonetheless, it could also be extended to include several other types of uncertainty, which are usually met in the real-life operation of contemporary urban networks. Such types of uncertainty may concern information acquisition and the day-to-day (or period-to-period) adjustment of the decision-making process of users. Other possible extensions refer to the more general case of multimodal networks with multiple-class users, in order to address issues related to the sustainable network development.

Acknowledgements

The authors are grateful to an anonymous reviewer who made several constructive and helpful comments on earlier version of this paper.

References


