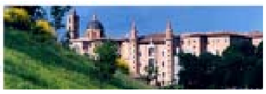

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Dynamic congestion pricing based on user-optimal stochastic learning models

Loukas Dimitriou[1] and Theodore Tsekeris[2*]

[1]*Department of Transportation Planning and Engineering, School of Civil Engineering, National Technical University of Athens, Greece*
[2]*Centre of Planning and Economic Research, Athens, Greece*

Abstract

This paper describes the development and implementation of a dynamic congestion pricing scheme, augmented with a game-theoretic evolutionary learning model, into a realistic urban highway in Athens, Greece. The model recognizes several important behavioral features related to the response of users to the congestion pricing strategy. Such features include day-to-day as well as within-day adjustment mechanisms in the departure time and route choice behavior of users with respect to changes in travel cost, and heterogeneous classes of road users. The results can provide useful insights concerning the implications of modeling the stochastic learning processes of users for the efficient deployment of dynamic congestion pricing schemes in urban highways. The study findings can facilitate the evaluation of the acceptability of alternative congestion pricing schemes by users and the revenue management of system operators.

Keywords: Dynamic congestion pricing; learning models; evolutionary discrete choice models; stochastic user optimal conditions; urban networks.

1. Introduction

The increased demand for passenger and freight mobility by road transport modes raises progressively the political and social awareness and acceptability for imposing congestion pricing schemes, in order to internalize the negative externalities of road traffic. First, public funding is insufficient to keep up with needs for building and maintaining roads. The principle of user-pays is now widely accepted and user charges are perceived as an important – if not essential – means of financing infrastructure. In addition, various road pricing schemes, particularly in relation to congestion pricing, are currently examined in Europe and elsewhere to help managing travel demand and optimizing the road capacity utilization in transport networks (De Palma et al., 2006). In contrast with other measures employed to charge road usage, such as gasoline taxes and flat (area, cordon or ring) tolls, which do not vary with time of day,

* Corresponding author: Theodore Tsekeris (tsek@kepe.gr)

dynamic congestion pricing allows the efficient use of road facilities or networks. In particular, the economic efficiency of road operations can be improved by imposing to the users of a road facility such a congestion toll that equals the difference between the marginal cost and the average cost of the marginal trip on that facility. This principle has been well developed in the economic literature since the appearance of the work of Pigou (1920). The dynamic congestion pricing takes into account the temporal structure of congestion in a road facility and allows imposing time-varying congestion tolls to the road users according to different time intervals of the day.

This paper describes the problem of dynamic congestion pricing based on the definition of marginal cost and it involves the interaction between a public highway authority and the group of road users at dynamic equilibrium network flow conditions. A key factor for designing and evaluating the success of alternative congestion pricing schemes refers to determining the adjustment mechanisms of users' responses to congestion tolls in both the short- and long-term horizon. These responses can involve changes in the amount of trip-making as well as the departure time and route choice behavior, dependent upon the characteristics of different user groups. Thus, the development of an optimal (fair) dynamic congestion pricing strategy should favorably incorporate appropriate methods for the discrete choice modeling of the evolutionary learning processes of users.

Based on the above theoretical requirements, this paper examines the design and evaluation of dynamic congestion pricing schemes in relation to the responses of users to congestion tolls within an evolutionary game-theoretic framework. This framework analyzes the day-to-day and within-day adjustment behavior of users to the selection of the optimal congestion tolls. The following section presents the theoretical background and existing approaches for the development of dynamic congestion pricing models. Section 3 describes the methodological approach adopted in the current study, which suitably combines an evolutionary game-theoretic learning model with a dynamic congestion pricing scheme. Section 4 provides the analysis of the results obtained from the proposed model and Section 5 concludes.

2. Theoretical framework and models of congestion pricing

Several congestion pricing models have appeared in the current literature and tested for various types of road facilities and toll configurations, based on different economic principles and performance objectives, such as those of the maximization of social surplus or profit, without constraints or subject to constraints related to social equity, spatial equity and elimination of queues (Yang and Zhang, 2002; De Palma et al., 2005). Despite that most of the existing analyses have employed static models for pricing simple road networks, an increasing number of studies recognize the need for capturing the temporal structure of demand and traffic conditions. The development of dynamic congestion pricing models allows considering the impact of time-varying tolling schemes on different (departure time, mode, route, destination) choices of users in networks where the link flow pattern changes within the day period.

Henderson (1974) showed that time-varying congestion tolls for a single bottleneck can significantly influence schedule delays and departure time decisions of travelers. Arnott et al. (1990) examined the effectiveness of static and dynamic (with time-varying and step toll) pricing policies in a parallel-route network. Carey and Srinivasan (1993) demonstrated the dependence of optimal congestion tolls on both the level and rate of change of traffic congestion and their importance on internalizing the congestion cost externalities in an example network. Huang and Yang (1996) used the optimal control theory for implementing time-varying congestion pricing into a parallel-route network. Wie and Tobin (1998) calculated the optimal time-varying tolls for every link of a network based on the theory of Marginal Cost Pricing (MCP). De Palma and Lindsey (2000, 2002) considered the problem of time-varying tolls in each link of a parallel-route network, based on the bottleneck model of Vickrey (1969). Wie (2007) considered the dynamic congestion pricing model to maximize the net consumer surplus when a public highway authority is constrained to impose congestion tolls only on a preselected subset of network links. By and large, these studies have indicated that time-varying congestion tolls result in higher efficiency gains than uniform tolls.

In addition to the issue of modeling congestion tolls by time of day, a range of additional elements is still lacking from most of the existing studies. Such elements refer to the heterogeneity of users in terms of their path cost perception and value of time, the dynamics of users' behavioral adjustments through learning processes from day to day as well as within day, and the lack of evaluating dynamic congestion pricing schemes at selected links of realistic urban or regional networks. Joksimovic et al. (2005) formulated the dynamic congestion pricing problem as a bi-level Mathematical Program with Equilibrium Constraints (MPEC) where congestion tolls are allowed to affect the (sequentially modeled) route and departure time choice of heterogeneous user groups within the day period. Zhao and Kockelman (2006) extended the model of Yang et al. (2004) in order to formulate the MCP problem, allowing the Stochastic User Equilibrium (SUE) assignment of demand onto a realistic regional network. Their study recognized group-based variations in the Value of Travel Time (*VOTT*) and it relaxed assumptions concerning the shape of time-varying congestion tolls across the intervals of the day.

Existing studies typically ignore the effect of users' memory and learning processes, which are conditional from historical (experienced) travel conditions, on origin–destination (O-D) travel demands and assume that users make their travel choices after the establishment of a stationary equilibrium state in the network. Nevertheless, the consideration of the day-to-day departure time and route choice reactions to dynamic congestion pricing schemes can allow better interpreting long-term changes in efficiency gains of congestion tolls over a certain planning horizon. The model proposed here builds on previous studies of MCP strategies (Yang et al., 2004; Zhao and Kockelman, 2006) in order to develop a dynamic MCP model based on the SUE with elastic demand and heterogeneous users. This model, which is analytically described in Section 3, is composed of two parts: (a) an evolutionary game-theoretic learning model and (b) a dynamic congestion pricing scheme based on the theory of MCP. The model implementation concerns a realistic scenario of congestion pricing of an urban highway passing through an untolled urban arterial network (see Section 4).

3. The proposed modeling approach

3.1 The evolutionary game-theoretic learning model

A discrete-time dynamic stochastic model based on the evolutionary game theory (Maynard Smith, 1982) is adopted here to represent behavioural adjustments of different temporal resolution, i.e. day-to-day and within-day changes in travel (departure time and route) choices. This model allows capturing the possibility that users make imperfect (sub-optimal) choices, as described within the framework of bounded rationality. The present modeling process provides an extension of the classical, non-cooperative game-theoretic formulation of the static SUE assignment to a repeated (evolutionary) game with 'memory'. In this model, the travel choice of each player (user) is made with respect to the history of path travel costs, which may refer to the corresponding interval of a past day or a preceding interval of the current day, as resulted by the choices made by all other players. The present study recognizes the heterogeneity among users by accounting for differences in: (a) the cost perception of alternative paths, (b) their *VOTT* and (c) the learning mechanisms used to make their travel decisions.

The representation of the dynamics of users' preferences when interacting with each other is described here by a composite dynamical system, which comprises two dynamical sub-systems in cascade. Consider that $C_{A,p}^{ij,t}$ is the cost of path $p \in P_{ij}$ between an O-D ($i-j$) traveled by a user belonging in class A at time interval t and $x_p^{ij,t}$ is the flow assigned on path p among $i-j$ pair during that interval. Then, the probability $\pi_{A,k,p}^{t}$ that path p is chosen by users of class A at interval t can be expressed as a discrete-time stochastic dynamical system of the following evolutionary logit logic:

$$\pi_{A,k,p}^{t} = \frac{\exp(-\theta^{t-k} C_{A,p}^{ij,t-k})}{\sum_{A}\sum_{p} \exp(-\theta^{t-k} C_{A,p}^{ij,t-k})} \tag{1}$$

where $k < t$ is a time interval whose path cost information is utilized by a user to choose path p at interval t and θ^{t-k} is the user's cost perception parameter at interval $t-k$. Moreover, another dynamical sub-system is formulated to account for the elasticity of the demand $d_A^{ij,t}$ of users belonging to class A moving between $i-j$ pair during interval t, through considering changes in the departure time choice in relation to the cost $C_A^{ij,t-k}$ experienced along the paths of the set P_{ij}, as follows:

$$d_A^{ij,t} = D_A^{ij,t} \exp^{a C_A^{ij,t-k}} \tag{2}$$

where $D_A^{ij,t}$ is the maximum (desired) demand for travel between $i-j$ and a is a scale parameter, which is here set equal to $a = -0.03$. By expressing the elasticity of demand with respect to path travel time in equation (2), the current sub-system can provide an adequate approximation of the users' responses in their trip departure time decisions, since it allows capturing the (re-)distribution of demand within the period-of-day in a recursive fashion. Nonetheless, other alternative models which express the departure time choice in relation to additional travel cost components, such as schedule delay costs, could also be used in the current framework.

The evolutionary models presented in equations (1) and (2) are cascade dynamical sub-systems which allow the loading of the network through taking into account different user classes, in terms of the information that users possess in making their travel (departure time and route) choices. The complete form of the dynamical learning system is as follows:

$$x_p^{ij,t} = \sum_A \sum_k u_A^{t-k} \, \pi_{A,k,p}^{t} \, d_A^{ij,t} \tag{3}$$

where u_A^{t-k} denotes the percentage of users of class A processing the information corresponding to interval $t-k$ regarding the cost of path p. The proposed stochastic evolutionary learning model provides an equilibrium-tending dynamical system, whose fixed point (attractor) is the stochastic user equilibrium. Such a system recognizes that the trajectory of the state of urban road networks moves towards equilibrium, but it rarely equilibrates in practice. This is because the traffic conditions required to ensure the system stability cannot be met in real-world situations.

Several assumptions can be adopted in order to express the distribution of users who process alternative sources of information for estimating the path cost. The present study employs a reinforcement learning logic, according to which a player (user) increases the frequency of an action (travel choice) when this action has given a relatively larger payoff than the other actions. Namely, the users learn, over time, to make better decisions (leading to lower realized travel costs) more often and worse decisions less often. The proposed methodological framework allows incorporating this learning system into the dynamic interplay between the decisions of the users and highway authority, which controls the time-varying congestion tolls. The following subsection describes the congestion pricing strategy of the highway authority to respond to the travel choices of users.

3.2 The dynamic congestion pricing sheme

In the field of public economics, marginal-cost pricing (MCP) of a good or service provision has been identified as the optimal (fairest) pricing strategy, wherein users are individually charged for their contribution to the total social cost. The MCP is also recognized, since the works of Walters (1961) and Vickrey (1969), as the first-best pricing strategy for internalizing the social cost of road usage to the individual travel cost. The dynamic road charging can address the congestion phenomena mainly experienced during rush hours in contemporary urban areas, meaning that congestion price discriminate with respect to congestion levels. Also, a fair congestion pricing should respect the pay-as-you-drive principle, implying that users are charged for the part of the network utilizing to execute a trip. Thus, the proposed MCP scheme is estimated here by imposing a time-varying toll to each link of the part of the network where the congestion pricing strategy will be implemented.

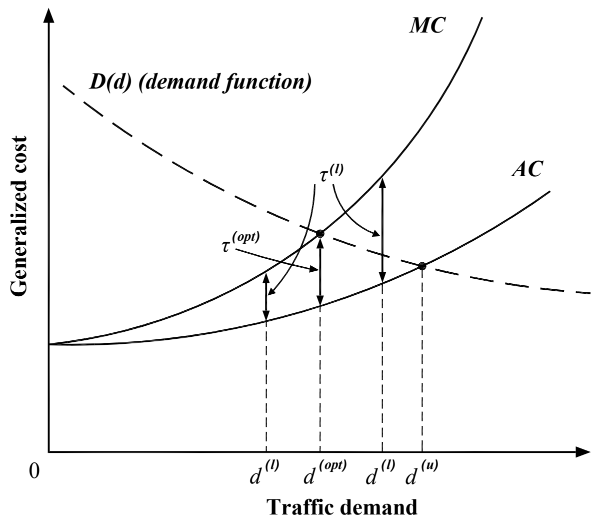

Figure 1: Single-link Average Cost (AC) and Marginal Cost (MC) *vs.* link traffic demand

Taking that f_m^t is the flow at link m during interval t and $c_m^t(f_m^t)$ is the corresponding average link cost (AC_m^t), the total cost (TC_m^t) of that link is equal to $TC_m^t = f_m^t\, c_m^t(f_m^t)$. Hence, the cost of the marginal user (one additional user entering the link) or the marginal cost (MC_m^t) (see Figure 1) is estimated as the derivative of the TC_m^t with respect to flow f_m^t, as follows:

$$MC_m^t = \left(TC_m^t\right)' = \left(f_m^t\, c_m^t(f_m^t)\right)' = c_m^t + f_m^t \left(c_m^t(f_m^t)\right)' = AC_m^t + \tau_m^t \tag{4}$$

where τ_m^t denotes the marginal cost of congestion. The average cost c_m^t at link m during interval t is composed of the value of travel time $VOTT_A$ of user class A and other costs g_m^t, which are irrelevant to the congestion price (for instance, flat toll fares), i.e. $c_m^t(f_m^t) = VOTT_A\; t(f_m^t) + g_m^t$, where $t(f_m^t)$ denotes the travel time at the specific link and interval. Provided that multiple user classes are identified with respect to their $VOTT$, the marginal congestion cost τ_m^t is weighted with the flow of users from different classes sharing link m:

$$\tau_m^t = \frac{\sum_{ijpA} x_p^{ij,t}\, \delta_{mp,A}^{ij,t}\; VOTT_A\; f_m^t\; t'_m(f_m^t)}{\sum_{ijpA} x_p^{ij,t}\, \delta_{mp,A}^{ij,t}} \tag{5}$$

where $x_p^{ij,t}$ refers to the path flow given by the evolutionary learning system of equation (3) and $\delta_{mp,A}^{ij,t}$ stands for a binary variable characterizing whether link m is included in path p connecting $i-j$ pair for user class A at interval t ($\delta_{mp,A}^{ij,t}=1$) or not ($\delta_{mp,A}^{ij,t}=0$), such that $\sum_{ijpA} x_p^{ij,t}\,\delta_{mp,A}^{ij,t} = f_m^t$. The following section presents the application of the proposed dynamic congestion pricing scheme, augmented with the evolutionary learning model, into a realistic part of the urban road network of Athens, Greece, where phenomena of increased congestion are observed in a daily recurrent fashion.

4. Experimental setup and results

The present application refers to a part of the urban network of Athens that is composed of primary and secondary roads, which are linked with a closed urban highway, called Attiki Odos. The study network (Figure 2) covers the most densely populated region along the highway, where the highest daily traffic volumes are observed. The network is composed of 54 links servicing the demand represented by a 10×10 O-D matrix. Attiki Odos operates under a Built-Own-Transfer (BOT) scheme by a Public-Private-Partnership (PPP), which imposes fixed toll charges (2.70 €/private car) to recover the investment costs.

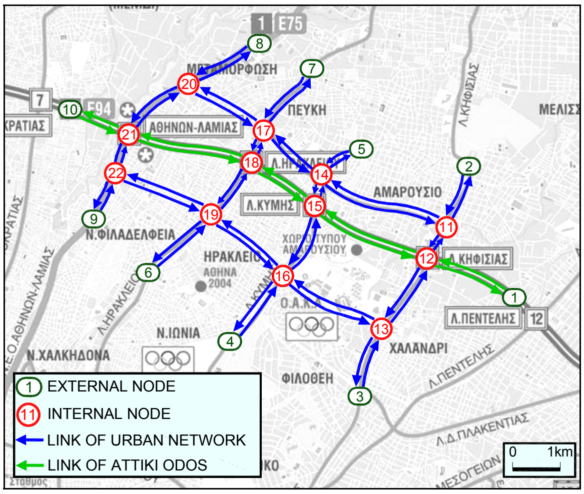

Figure 2: Configuration and coding of the urban network and tolled highway of the study

The deployment of the dynamic congestion pricing strategy is investigated here for the links of the urban highway (coded in green). The analysis identifies two *VOTT* user classes, based on their travel purpose and income level. The first user class (VOTT class 1) has an hourly $VOTT = 4$€, representing commuters, work-based trips and travelers of increased income, while the second user class (VOTT class 2) has an hourly $VOTT = 1.5$€, representing low income travelers and more elastic trips. The users belonging to the first class are selected as the 80% of the total traveler population. In addition, the users are distinguished with respect to the mechanism employing for processing information about their path travel cost. The users of the first information class (Info class 1) rely on historical information concerning their travel cost, while the users of the second information class (Info class 2) have access at the beginning of their trip to reliable and real-time path cost information. Although the model proposed in equation (3) can consider time-varying composition of user information classes, the present analysis assumes this composition to be constant, selecting as the 80% of the total traveler population to rely on historical path cost information. The discrete-time dynamical system represents the learning process with a time step equal to 15 min. Hence, it holds that $k=96$ for the users of the Info class 1 and $k=1$ for the users of the Info class 2. The calculation of the various elements of link cost is based on the Bureau of Public Roads (BPR) formula:

$$t_m(f_m) = t_m^0\left(1+\mu\left(\frac{f_m}{G_m}\right)^{\beta}\right), \qquad \forall\, m \in M \tag{6}$$

where t_m^0 is the free flow link travel time and G_m is the maximum capacity at link m and parameters $\mu = 0.15$ and $\beta = 4$. By adopting the BPR formula, τ_m^t is calculated as follows:

$$\tau_m^t = \frac{\sum\limits_{ijpA} x_p^{ij,t}\ \delta_{mp,A}^{ij,t}\ VOTT_A\left(\mu\,\beta\, t_m^0\left(\frac{f_m^t}{G_m}\right)^{\beta}\right)}{\sum\limits_{ijpA} x_p^{ij,t}\ \delta_{mp,A}^{ij,t}}, \qquad \forall\, m \in M \tag{7}$$

Figure 3 illustrates the diurnal fluctuations of the demand for travel along the tolled highway during a typical day period, as it is estimated from the evolutionary learning system. These temporal variations reflect changes in the level of travel demand along the highway due to dynamic adjustments in departure time and route choices of users to the dynamic congestion pricing policy. These adjustments can result in shift of the trip demand to other (with smaller perceived cost) intervals of the day period or diversion of a proportion of traffic towards the untolled links of the urban network. Figure 4 depicts the temporal evolution of the dynamic congestion tolls at each link, i.e. link 12-15, link 15-18, and link 18-21, along the east-west direction of the tolled highway during the same day period. The comparison between Figures 3 and 4 indicates the significant sensitivity of travel demand to the time-varying congestion charges across the day period. The price of congestion tolls is relatively small and basically similar during the low-demand periods of the day. On the contrary, the congestion charges manifest considerable spatial variations among the tolled links during the high-demand periods of the day, i.e. between 8:00 am and 10:00 pm.

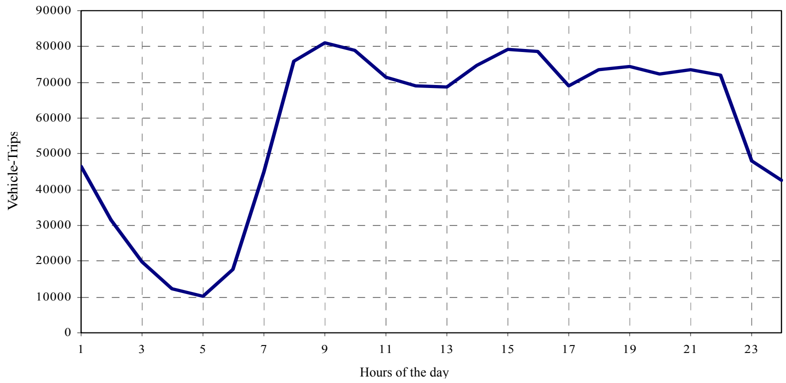

Figure 3: The temporal evolution of the estimated travel demand along the highway during a typical day period

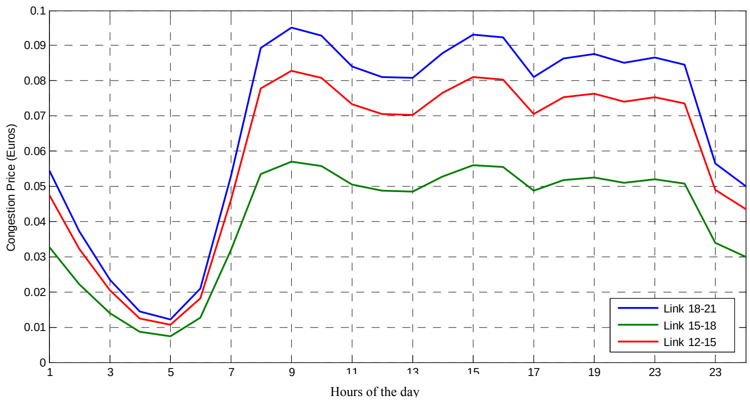

Figure 4: The temporal evolution of the dynamic congestion toll prices at each link along the east-west direction of the highway during a typical day period

Table 1 presents the congestion charges and *VOTT* for the links of the east-west direction of the highway at the time of maximum demand, during the morning rush hour (9:00 am). At the specific time and part of the highway, each user belonging to *VOTT* class 1 will spend a total of 1.32+0.85+1.34=3.51 min with *VOTT* equal to 3.51 min × 0.066 €/min = 0.23€, while the imposed congestion charge is equal to 0.082+0.057+0.095=0.234€. Hence, the congestion charge in this case approximately amounts to the 102% of the value of travel time and the 9% of the fixed toll charge. Correspondingly, each user belonging to *VOTT* class 2 will have *VOTT* equal to 3.51 min × 0.025 €/min = 0.08 €, and a congestion charge (0.234€) which approximately amounts to the 293% of the value of travel time.

Table 1: Congestion charges and $VOTT$ for the links of the east-west direction of the highway at the time of maximum demand (9:00 am)

Link	Length (m)	Travel time (min)	Congestion charge (€)	Value of travel time (€)
12-15	1960	1.32	0.082	0.081
15-18	1270	0.85	0.057	0.052
18-21	1990	1.34	0.095	0.083

These results signify the potential influence of time-varying congestion tolls on the travel (departure time and route) decisions of heterogeneous road users. In particular, they demonstrate the importance of evaluating the within-day effects of dynamic congestion pricing on travel demand, in terms of the level as well as the temporal spreading of congestion during peak travel periods. The need for considering the evolutionary learning processes and their effect on the departure time and route choice behavior of users is magnified by the configuration of the given network, where congestion charging is implemented into a limited number of links, i.e. those of the urban highway.

5. Summary and conclusions

This paper presented the development and implementation of a dynamic congestion pricing scheme, based on the concept of marginal-cost pricing. The model incorporates a number of important aspects of user behavior, which are not typically considered in the design and evaluation of congestion pricing strategies. Specifically, it models the day-to-day and within-day adjustments of their departure time and route choice behavior through employing a game-theoretic evolutionary learning model. Also, the proposed framework enables the representation of additional elements of the user decision-making mechanism, such as their heterogeneity in path cost perception and the value of time. The model is implemented into a region of Athens, Greece, where the tolled infrastructure refers to a limited number of links of a highway crossing a toll-free urban network.

The results demonstrate the significance of modeling the evolutionary learning process of users in order to determine their departure time and route choice responses to the imposed congestion charges. In particular, both the level and spatio-temporal distribution of demand are found to be sensitive to the congestion toll prices. The congestion charges can be regarded as considerable travel expenditure for the users of the highway, in relation to the fixed toll price in the specific area, while they are larger than the value of travel time for all user classes. Therefore, the public highway authorities should take into account such behavioral features of users in order to achieve the efficient deployment of dynamic congestion pricing and improve the toll revenue management. Finally, the study findings signify the importance of adopting time-varying congestion charging to allow differentiation between peak and off-peak road usage and enhance the public acceptability to such schemes.

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