Elastic Multi-User Stochastic Equilibrium Toll Design with Direct Search Meta-Heuristics

Loukas Dimitriou[1] and Theodore Tsekeris[2]

Abstract
This study describes the use of a Direct Search (DS) meta-heuristic algorithm for solving the toll design problem, in terms of finding the optimum toll level, in private roads. The problem is formulated as a nonconvex, bilevel nonlinear mathematical program which seeks to maximize toll revenues while taking into account the travel responses of network users, through a multi-class stochastic user equilibrium traffic assignment model with elastic demand. The algorithm is implemented onto a real-life urban sub-network which includes a private highway. The results show the ability of the DS algorithm to relatively quickly find an optimal solution and signify its potential to provide a competitive alternative to the currently used genetic algorithm (GA) approach for solving such types of nonconvex bilevel programs in the sector of road transport services.

Keywords: Toll Design, Stochastic Equilibrium Traffic Assignment, Urban Networks, Meta-Heuristics, Direct Search, Genetic Algorithms.

1 Introduction

In contemporary metropolitan areas, road pricing is increasingly considered as a promising policy measure to manage travel demand, and reduce traffic congestion and environmental pollution. Moreover, road pricing can be seen nowadays as a tool for levering the economic efficiency of the urban transport system, through ensuring the appropriate funds for road investment decisions, such as capacity expansion, regular maintenance, and improved public transport services. Recent developments in electronic payment mechanisms offer lower cost and new possibilities for road pricing systems.

The majority of existing studies investigate road pricing based on the marginal (or 'first-best') pricing principle. This principle implies a toll that is equal to the difference between the marginal social cost and marginal private cost and is charged on each link of the network in order to maximize social welfare [1]. However, in most real-life applications, the marginal cost is impossible to be imposed, due to several factors not practically controllable by the system manager (or service provider). Such factors include the unwillingness of the general public to pay tolls in congested urban roads on the basis of the marginal cost-pricing scheme, and the high costs of setting up and maintaining toll facilities at a large number of links in real urban networks.

In reality, only a very small set of links (e.g., highway segments, bridges and tunnels) is tolled in a transport network, surrounded by untolled alternative links. Thus, a more realistic version of the toll design problem refers to the so-called 'second-best' pricing, for the optimum design of toll services in only a subset of network links [2, 3]. In practice, these services are usually designed by Private Public Partnerships (PPPs) under Build-Operate-Transfer (BOT) concession schemes, which refer to the construction and operation of specific links over a transport network by a private firm (private-sector operator) for a given period of time sufficient to attain an agreed level of investment benefit. This study adopts the design strategy of the firm, which is oblivious to social welfare and aims at maximizing its own revenue (profit), in order to payback the investment costs and afford the operation and maintenance costs of the road services.

The firm's goal of revenue maximization is commonly subject to specific constraints to the lower and upper allowable toll levels, while it takes into account the response of network users. The modeling of users' responses encounters network spillover effects on the untolled roads, in conjunction with the effect of traffic volumes on the toll roads. In addition to the above assumptions, this study considers the existence of multiple user groups in order to address the heterogeneous characteristics of travelers with different trip purposes and income levels. Further information about the settings of the toll design problem is provided in Section 2.

This paper adopts a Stackelberg game-theoretic, bi-level programming formulation of the toll design problem, where the 'leader' (in the upper-level problem) is the private operator and the 'follower' (in the lower-level problem) refers to network users (see Section 3). In order to address the complex, non-convex features of the toll design problem, this study suggests the novel application of a Direct Search (DS) meta-heuristic solution method (see Section 4), in addition to a genetic algorithm (GA) approach, which is broadly used for solving bi-level optimization problems in transport planning. Section 5 describes numerical results based on a real-world network application and discusses the potential usage of the DS as a competitive alternative to the GA for handling such types of complex, bi-level programming problems in road toll (and other transport sector)

[1] Loukas Dimitriou is with the Department of Transportation Planning and Engineering, National Technical University of Athens, Athens, Greece (email: lucdimit@central.ntua.gr)

[2] Theodore Tsekeris is with the Centre of Planning and Economic Research, Athens, Greece (email: tsek@kepe.gr)

services. Section 6 summarizes and concludes the findings of the study.

2 Description of the toll design problem

The problem of optimum toll design involves the determination of the optimal toll levels at the links of a road network, while accounting for the responses in the (route choice and/or trip-making decision) behavior of travelers. A large number of alternative modeling formulations has been hitherto proposed in the literature for solving the toll design problem. Most of these studies (e.g., see [4-7]) typically assume that all travelers have perfect information regarding the exact cost of traveling on every route of the network and that they all behave identically, choosing the route with the least cost between their origin and destination (O-D) pair.

On the contrary, the modeling of variations in the perceived generalized travel cost of users and their uncertainty in route choice behavior can provide a more realistic network assignment assumption, giving rise to the stochastic user equilibrium (SUE) assignment of travel demand [8]. The SUE implies a network condition where no traveler can improve his/her perceived travel time by unilaterally changing routes. The studies of Smith et al. [9] and Yang [10] considered the marginal cost pricing under logit-based SUE. Also, Chen et al. [11] employed the logit-based SUE for finding the optimal link-toll pattern of the network that would induce the lowest total travel cost (exclusive of tolls).

The expression of the elasticity of demand with respect to path travel cost can provide a more adequate approximation of users' responses in their trip-making, adding to the route choice the binary choice of whether they will make a trip or not. Also, the modeling of the value of travel time (VOTT) among travelers of different income and trip purpose may account for variations in route choice behavior due to varying tradeoffs between money and time when simulating their response to toll charge. As it is typically adopted in the literature, this study assumes that, for each user group, travelers share the same discrete VOTT probability distributions [12]. The use of multi-class traffic assignment procedures with elastic demand has been mostly considered for the case of marginal cost pricing [13, 14], and, at a lesser extent, for second-best pricing schemes which seek to minimize the total network travel time [15].

On the contrary, the present study focuses on the revenue-maximizing second-best pricing strategy, which is widely met in the toll services of private highways in several countries around the world, as developed by PPPs under BOT schemes. Such an optimum toll design strategy was examined in [16], assuming deterministic route choice behavior with fixed demand and homogeneous users. The optimum toll design strategy proposed here incorporates the SUE assignment of elastic travel demand with multi-user groups. It concerns a realistic scenario of the pricing of a private highway passing through an untolled urban arterial network (see Section 5). The present problem formulation (see Section 3) is expressed in a generalized form so that allow accommodating alternative pricing tactics, e.g., of uniform or entry-based toll charges [17].

3 Bilevel program formulation

Bilevel programming offers a framework for modeling situations where one player (the 'leader') integrates within its design strategy the reaction of a second player (the 'follower') to its own course of action. Bilevel problems are closely related to static Stackelberg games and mathematical programs with equilibrium constraints (MPEC), in which the lower-level solution characterizes the equilibrium state of the system. The optimum toll design problem in a private highway can be formulated as a bi-level program, where the private-sector operator determines the toll level at the upper level, possibly subject to constraints imposed by a regulatory authority, and, then, at the lower level, the users of different groups react by choosing alternative routes, based on a multi-class SUE assignment model. The present multi-class SUE assignment with elastic demand is formulated as an unconstrained minimization problem [14]. In this study, it is assumed that the set (number and location) of tolled highway links, which constitute part of an urban network, has already been identified before the toll-design problem is formulated and solved.

Consider a network $G(N, A)$ composed of a set of N nodes and A links, which connect the origin zone r with destination zone s, and q_m^{rs} be the demand of the users of group m for moving between O-D pair r-s. Also, consider the link travel time function $t_a(x_a)$ as being positive and monotonically increasing with traffic flow x_a at each link $a \in A$. Then, the bilevel optimal toll and multi-class SUE traffic assignment problem with elastic demand can be expressed as follows:

Upper-level problem:

$$\max_{p,e} R(p_i, e_{im}^{rs}) = \sum_{rsikm} p_i \left(\delta_{i,km}^{rs} e_{im}^{rs}\right), \ \forall\, i \in E \qquad (1)$$

subject to: $$p_{\min} \le p_i \le p_{\max} \qquad (2)$$

Lower-level problem:

$$\min_{x,q} Z(x,q) = \sum_{rsakm} \mathrm{VOTT}_m \, \delta_{a,km}^{rs} \, t_a(x_a) \, x_a - \sum_{rsakm} \mathrm{VOTT}_m \, \delta_{a,km}^{rs} \int_0^{x_a} t_a(w) \mathrm{d}w +$$

$$\sum_{rsm} \text{VOTT}_m D_{rsm}^{-1}(q_m^{rs}) D_{rsm}\left(S_m^{rs}(x)\right) -$$
$$\sum_{rsm} S_m^{rs}(x) D_{rsm}\left(S_m^{rs}(x)\right) +$$
$$\sum_{rsm} \int_0^{q_m^{rs}} \text{VOTT}_m D_{rsm}^{-1}(q)\mathrm{d}q -$$
$$\sum_{rsm} q_m^{rs}\, \text{VOTT}_m D_{rsm}^{-1}(q_m^{rs}) \quad (3)$$

where R denotes the revenue function of the private-sector highway manager, p_i is the toll charge imposed on the users entering the highway from access node $i \in E$, with $E \subset N$, e_{im}^{rs} is the volume of users of group m moving between r - s pair and entering the highway from access node i, and $\delta_{i,km}^{rs}$ is a binary variable which takes the value 1, if node i is part of the path $k \in K^{rs}$ followed by users of group m between r - s pair, or the value 0 otherwise, with K^{rs} being the set of feasible path flows among that pair. The scalars $p_{\min}$ and $p_{\max}$ are the minimum and maximum toll levels. At the lower-level problem, Z expresses the objective function of network users of different groups m, with corresponding value of travel time VOTT_m, who seek to minimize their perceived generalized travel cost, assuming that they have elastic demand and their route choice behavior is consistent with the SUE link flow conditions. The binary variable $\delta_{a,km}^{rs}$ takes the value 1, if link a is part of the path $k \in K^{rs}$ followed by users of group m between r - s pair, or the value 0 otherwise.

Assuming that the demand function D_{rsm} is nonnegative and strictly decreasing in own-path cost, then, $q_m^{rs} = D_{rsm}(S_m^{rs})$ and $S_m^{rs} = D_{rsm}^{-1}(q_m^{rs})$, where D_{rsm}^{-1} is the inverse demand function and S_m^{rs} is the perceived travel cost function. The latter function is expressed in relation to the expectation E of the total path travel cost C_{km}^{rs} between r - s pair for user group m, as follows:

$$S_m^{rs}(x) = E\left[\min_{k \in K^{rs}} \left\{C_{km}^{rs}\right\} \middle| \, C^{rs}(x)\right], \quad (4)$$

$$\frac{\partial S_m^{rs}(C^{rs})}{\partial C_{km}^{rs}} = P_{km}^{rs}, \quad (5)$$

where P_{km}^{rs} denotes the probability that users of group m select path k for moving between r - s pair. The multi-class SUE assignment model allows capturing the possibility that users of all groups make imperfect (sub-optimal) choices, as described within the framework of bounded rationality. Then, the measure of probability P_{km}^{rs} depends on the following utility function:

$$U_{km}^{rs} = -\theta C_{km}^{rs} + \varepsilon_{km}^{rs}, \quad (6)$$

where U_{km}^{rs} expresses the utility of users of group m selecting path k for moving between r - s pair, θ is the perceived path cost parameter and ε_{km}^{rs} is a random error term, independent and identically distributed (iid) for all routes, which follows here a Gumbel distribution, hence, yielding a logit model.

The path travel cost C_{km}^{rs} is expressed in monetary terms, as a composite function of the value of travel time and toll charge:

$$C_{km}^{rs} = \sum_a \text{VOTT}_m \delta_{a,km}^{rs} t(x_a) + \delta_{i,km}^{rs} p_i \quad (7)$$

The estimation of the response of users' demand to changes in path travel cost is based here on the following relationship [18]:

$$D_{rsm}^{(n)} = D_{rsm}^0 \exp(uC_{rs}), \ \forall\, r,s,m, \quad (8)$$

where D_{rsm}^0 refers to the initial demand level of users of group m moving between r - s pair and u is a scale parameter (here, a relatively large value of $u = -0.3$ is used, since the path cost is expressed in monetary terms). In order to calculate the travel time t_a at link a, the well-known Bureau of Public Roads (BPR) function is used, as follows:

$$t_a(x_a) = t_a^0\left(1 + \mu\left(\frac{x_a}{G_a}\right)^\beta\right), \ \forall\, a \in A \quad (9)$$

where t_a^0 is the link travel time at free-flow conditions, μ and β are parameters referring to local operating conditions (in this study, $\mu = 0.15$ and $\beta = 4$) and G_a is the maximum traffic capacity at link a.

The present bilevel programming problem seeks to maximize the volume of users entering the highway from various access points, taking into account the impact of toll level on the SUE conditions and travel demand of different user groups at the whole network. As typically any other type of bilevel (or MPEC) formulation, it is intrinsically nonconvex and complex in its nature, due to the nonlinear formulations of the functions involved in the upper and lower-level problems. The nonlinearity and nonconvexity portend the existence of local solutions, which imply that it might be difficult to solve for a global optimum (or adequately near-optimum) solution. In view of the difficulty in applying the standard algorithmic approaches for search of the global optimum, this study adopts a Direct Search meta-heuristic method, which is described in the following section.

4 Solution procedures

The majority of current studies examining the bilevel toll design and traffic assignment problem have employed analytical derivative-based methods, such as the sensitivity-analysis-based (SAB) algorithm [6], the feasible direction method and the interior point algorithm [11], Lagrangian multiplier and other heuristic methods [7, 19]. These algorithms have been generally found to be incapable of reaching globally optimal solutions, very computationally intensive and practically intractable for realistic-size problems. In order to address this intractability, Roch et al. [16] applied a bound approximation method, considering upper bounds for toll revenues. Earlier attempts include the application of genetic algorithms (GAs) [20, 21] and a penalty function approach using a simulated annealing method [15].

Successful implementations of algorithms for solving bilevel programs rely largely on their ability to adapt to the particular nature of the problem under consideration and handle realistic complications. A group of such algorithms refers to Direct Search (DS) techniques, which are derivative-free, global optimization methods. These meta-heuristic techniques were first introduced by Hook and Jeeves [22]. They are hybrid methods that use information about the model performance, based on multiple trial values of the control variable(s), in order to generate search directions.

Hence, their usage is particularly suitable in cases where application of methods based on analytical derivative information is practically not feasible. The DS algorithms typically converge to adequately (near-) optimal solutions, through employing a wide range of search mechanisms to explore the solution space (see [23]). A Generalized Pattern Search (GPS) method is adopted here [24, 25], where search directions are generated using a pattern search mechanism.

The steps of the solution of the bilevel toll design and traffic assignment problem using the DS technique are described in Table 1. The basic concept of the iterative solution procedure is to evaluate the model performance, i.e., find the toll revenues, given a specific toll level (control variable of the problem), through solving the lower-level problem, to estimate users' responses with the multi-class SUE traffic assignment with elastic demand. Then, a new optimal toll level is found at the upper-level problem through using a pattern search with a set of exploratory moves.

Table 1: Description of the steps of the DS solution procedure of the bilevel toll design and traffic assignment problem

Step 1. (Initialization)
Produce an initial search pattern and select the properties of the Direct Search mechanism

DO UNTIL CONVERGENCE:

Step 2. (Lower-level problem)
Produce a new SUE link flow solution for each search pattern element.

Step 3. (Performance evaluation)
Estimate the toll revenues for each element of the search pattern.

Step 4. (Upper-level problem)
Estimate the optimum toll level by performing a new pattern search.

Specifically, the GPS method starts with an initial guess, $p_0 \in \Re^n$, of the optimum toll level, an initial search pattern, $\Pi_0 \in \Re^{2n \times n}$ (expressed in a matrix form), and a scalar parameter, $\Delta_0 > 0$, referred to as mesh size, which controls the step size (length) of the pattern search. An iterative procedure is then followed to update Δ_κ and Π_κ at each iteration κ in a random process of exploratory moves until reaching the optimum toll level. Hence, in the present case of maximizing the toll revenues, the following general scheme is used:

For $\kappa = 0, 1, \ldots$
DO UNTIL Convergence:
1. Find a set of feasible solutions P^κ using exploratory moves ($\Delta_\kappa, \Pi_\kappa$)
2. *IF* $R(P_j^{\kappa+1}) > R(P_{opt}^\kappa)$ *THEN* $P_{opt}^{\kappa+1} = P_j^{\kappa+1}$;
ELSE $P_{opt}^{\kappa+1} = P_{opt}^\kappa$
3. Update (Δ_κ, Π_κ)

In order to perform the pattern search, the set of ν feasible solutions at each iteration κ is structured as matrix $P^\kappa = (P_1^\kappa, \ldots, P_j^\kappa, \ldots, P_\nu^\kappa)^{\mathrm{T}}$, where $P_j^\kappa = (p_1, p_2, \ldots, p_n)$ is the vector of the values of the n variables of a feasible solution. Various heuristics can be employed to produce exploratory moves. In this study, a set of $2n$ exploratory moves, with n be the number of control variables (in the present case, $n = 1$), is carried out in each iteration κ, hence, $\nu = 2n$. These exploratory moves are produced through unilateral alterations of the control variable in both coordinate directions, while keeping the search pattern Π constant, in order to approximate improved solutions, using the following equation:

$$s^\kappa = \Delta^\kappa \times \Pi , \tag{10}$$

where

$$\Pi = \begin{bmatrix} \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & & & \\ 0 & 0 & \dots & 1 \end{bmatrix}_{n\times n} \\ \hline \begin{bmatrix} -1 & 0 & \dots & 0 \\ 0 & -1 & \dots & 0 \\ \vdots & & & \\ 0 & 0 & \dots & -1 \end{bmatrix}_{n\times n} \end{bmatrix}_{2n\times n}$$

At each iteration κ, the matrix of feasible solutions to be searched is produced as follows:

$$P^{\kappa} = H^{\kappa} + s^{\kappa}, \quad (11)$$

where H^{κ} is an auxiliary matrix of order $n\times 2n$, composed of the best solutions obtained so far, i.e., $H^{\kappa} = (\mathrm{P}_{\mathrm{opt}}^{\kappa(1)}, \mathrm{P}_{\mathrm{opt}}^{\kappa(2)}, ..., \mathrm{P}_{\mathrm{opt}}^{\kappa(2n)})^{\mathrm{T}}$. Namely, the pattern search is made on the values of the variables with the best performance, in terms of the objective function.

At every successful iteration where a new value of toll revenues, $R(P_j^{\kappa+1})$, is found, such that $R(P_j^{\kappa+1}) > R(P_{opt}^{\kappa})$, the search step length is increased by a rate of 2, i.e., $\Delta_{\kappa+1} = 2\times\Delta_{\kappa}$, to facilitate the acceleration of convergence. At every unsuccessful iteration, the mesh (s_{κ}) is contracted by a rate of $1/2$, i.e. $\Delta_{\kappa+1} = 1/2\times\Delta_{\kappa}$, in order to decrease the search step length and, hence, perform a more detailed search in the currently explored area. The specific GPS is known as Generating Set Search (GSS) method [26].

It is noted that, in the current case of revenue maximization with minimum and maximum toll constraints, the exploratory moves should always lie in the feasible region. The convergence analysis of the algorithm used here has been investigated in [24], [25] and [26] for both the cases of constrained and unconstrained optimization. As far as the initial conditions of the solution procedure is concerned, the initial parameter value is set equal to $\Delta_0 = 1$, while an initial value of $p_0 = 5$ € is used for the tolls. The DS algorithm is considered to converge when the mesh size Δ_{κ} reaches a very small value, which is set here equal to 10^{-6}, or a maximum number of 100 total iterations has been performed.

For comparison purposes, a genetic algorithm (GA) is also tested and evaluated here for the solution of the bilevel toll design problem with multi-class SUE assignment and elastic demand. The basic concept used in solving the given bilevel programming problem with the GA is similar to that of the solution procedure followed with the DS technique (see Table 1). Table 2 describes the steps of the GA solution procedure. The GA operations (selection, crossover and mutation) resemble the natural processes of the evolution of a population (see [27]).

Table 2: Description of the steps of the GA solution procedure of the bilevel toll design and traffic assignment problem

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Step 1. (Initialization)
Code the control variable to a finite string, map the revenue function to a fitness function and randomly select an initial population.

DO UNTIL CONVERGENCE:

Step 2. (Lower-level problem)
Produce a new SUE link flow solution for each individual of the GA population.

Step 3. (Performance evaluation)
Estimate the toll revenues for each individual of the GA population.

Step 4. (Upper-level problem)
Produce a 'genetically' improved population through the selection, crossover and mutation operations to obtain a new optimum toll level.
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In this study, the coding of the control variable p is based on a binary coding scheme. The GA is composed of 20 individuals and the reproduction operator employs a tournament selection between 3 candidate individuals. The crossover operation uses a relatively high crossover rate equal to 75%, augmented with an elitism strategy, which enhance the probability of the selected individuals to exchange genetic information. A mutation rate equal to 5% is used to diminish the probability of finding a false peak. The convergence of the GA is considered to be achieved when the average of the population performance will not be significantly improved for 10 successive generations, or a maximum number of 50 generations has been performed.

5 Computational experiments

The present application refers to a part of the urban road network of Athens, Greece, that is composed of primary and secondary roads, which are linked with a closed urban highway, called Attiki Odos. The study network (Figure 1) covers the most densely populated region along the highway, where the heaviest daily traffic volumes are observed. The network is composed of 54 links servicing the demand represented by a 10×10 O-D matrix. Attiki Odos operates under a BOT concession scheme by a PPP, which imposes fixed toll charges (2.70 €/private car) to recover the investment costs and cover operating and maintenance costs. The current design process adopts a commonly-used flat toll policy, where the same toll charge is imposed on the users of all links at each access point, as in the present case of Attiki Odos. The optimum toll level is investigated for the private highway links (coded in green) during the morning peak hour 8:00-9:00am.

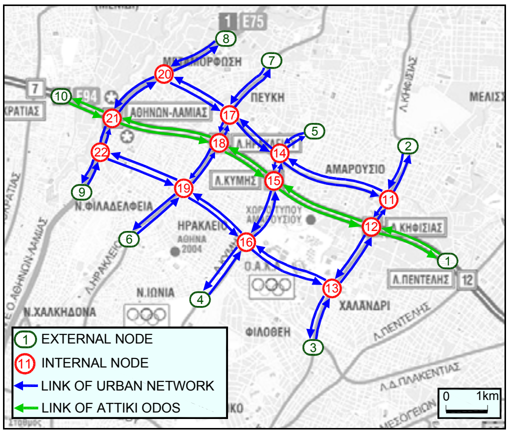

Figure 1: Configuration and coding of the urban network and tolled highway of the study

The present study identifies two VOTT user groups, based on their travel purpose and income level. The first VOTT group has an hourly VOTT=4 €, representing commuters and travelers of increased income, while the second group has an hourly VOTT = 1.5 €, representing more elastic trips and low income travelers. The minimum and maximum toll levels are set equal to $p_{\min}$=0 € and $p_{\max}$=7 € respectively.

The numerical results are evaluated with regard to the ability of the solution procedures to maximize toll revenues as well as the efficient use of computational resources. Both the DS and GA methods are found to work well, in terms of finding an optimum toll solution, although the considerable size of the given real-world problem. Figure 2 and Figure 3 indicate the convergence behavior of the DS algorithm in terms of the performance of the objective function R and the mesh size Δ_κ respectively.

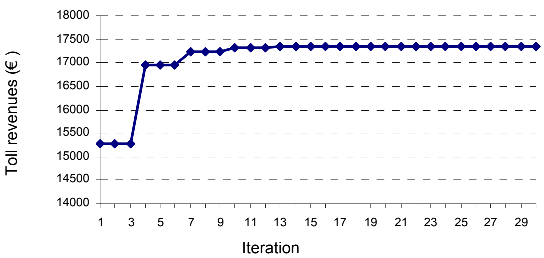

Figure 2: Convergence of the DS algorithm

The final solution of the DS algorithm results in a toll level equal to p=2.39 €, yielding a maximum revenue equal to R=17348 € (see Figure 2). The DS algorithm is found to converge within an average period of 40 min, using a total number of 60 evaluations of the lower-level problem (i.e., 2 exploratory moves × 30 iterations), namely, each iteration requires approximately 40 sec of CPU time, using a common PC facility with modest processing capabilities. The mesh size, Δ_κ, which denotes the step size of the pattern search at each iteration, is not found to exhibit considerable changes after the first 15 iterations (see Figure 3).

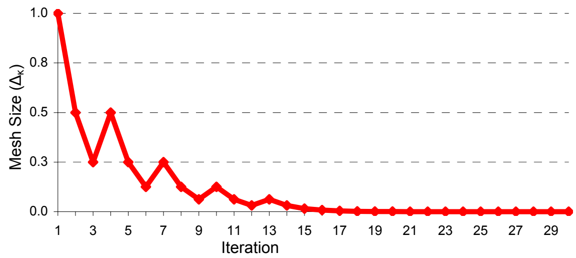

Figure 3: Change of mesh size, Δ_κ, for each iteration

Figure 4 presents the convergence of the GA, in terms of the best value and mean (population average) value of the objective (fitness) function. The final solution of the GA results in an optimum toll level and maximum revenues similar to those obtained from the DS algorithm, that is, p=2.39 € and R=17348 €. However, the GA solution appears to be much more expensive in computational cost than the DS algorithm. Specifically, the GA is found to converge within an average period of 650 min, using an average number of 50 generations, namely, each generation requires approximately 13 min of CPU time (i.e., 40 sec for each individual of the GA population).

In terms of their convergence effort, each iteration of the DS algorithm is about 10 times more computationally efficient than the GA generation. The increased computational cost of the GA, in comparison to that of the DS method, can be attributed to the fact that, given the narrowness of the feasible search area (between 0 € - 7 €), it can relatively fast identify a near-optimal solution from the initial population and, then, fairly exhaust its ability to search globally, concentrating on small local regions to finely tune the optimal p value.

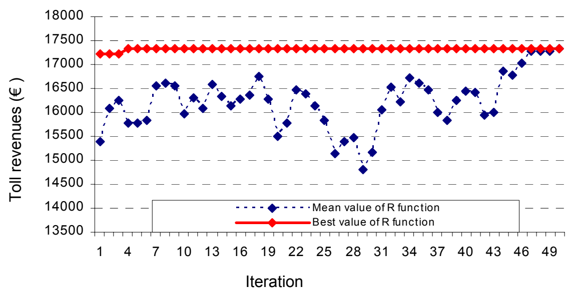

Figure 4: Convergence of the Genetic Algorithm

The solutions obtained from the DS and GA approaches are significantly dependent upon the assumptions adopted in the lower-level problem, concerning the representation of users' travel responses to the imposed toll level. Despite that the DS is able to provide optimal solutions for alternative settings of the given problem (see Section 2), it is of particular interest here to study the impact of a different traffic assignment logic on the resulting toll level and revenues.

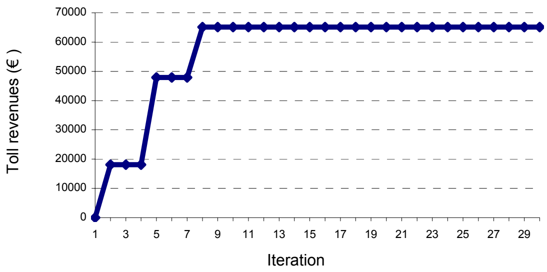

Figure 5: Convergence of the DS algorithm for the inelastic single-user class SUE assignment

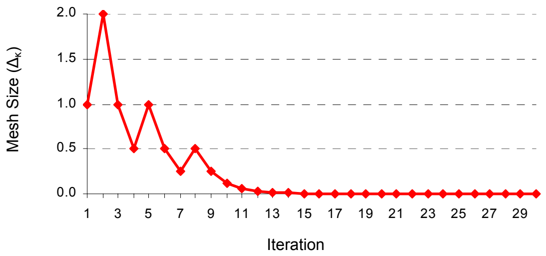

Figure 6: Mesh size, Δ_κ, for each iteration for the inelastic single-user class SUE assignment

Figure 5 and Figure 6 illustrate the solution of the toll design problem with a single-user class SUE assignment and inelastic demand (i.e., $u = 0$ in equation (8)), in terms of the performance of the R function and the mesh size Δ_κ respectively. The DS algorithm is found to provide an optimum toll level which is equal to its maximum allowable value, i.e., p =7 €, and total revenues R =65118 € (see Figure 5). These results make obvious that the use of a simplified traffic assignment model, such as that of the single-class SUE with inelastic demand, can lead to significantly higher toll level and revenues. These differences are due to the limited ability of such a simplified assignment procedure to capture the full range of users' responses, in terms of their trip-making decision and different money and time tradeoff mechanisms, to the imposed toll level.

Also, the usage of a different type of network assignment model is found to lead to changes in the computational performance of the DS algorithm. These changes can be explained by the fact that the simplified treatment of the SUE traffic assignment makes the solution procedure require a much less computational effort (about 1/10 of CPU time) for each lower-level problem evaluation and, hence, faster converge to an optimum toll level solution.

6 Conclusions

The results signify the potential of applying the current Generating Set Search (GSS) method, and, possibly, other germane DS techniques, into the design of efficient private road pricing strategies in realistic-size urban networks. The DS algorithm was found to provide a feasible alternative approach to the GA method, which is currently used for the solution of such bilevel programs as that of the optimum road toll design. Particularly, the DS was found to be much more computationally efficient than the GA, while both algorithms converge to the same optimum solution. This fact demonstrates the potential ability of the DS technique to efficiently handle computationally intensive problems in realistic-size applications, which may further involve CPU time burdens. Furthermore, the present findings stress the importance of the selection of the appropriate traffic assignment model that represents users' responses on the outcome of the solution procedure.

Also, the current promising results encourage future applications of DS techniques into other bilevel programs, which may concern different problems in the sector of road transport services, such as those of optimal road and public transport network design, transport infrastructure maintenance scheduling and capacity expansion, traffic signal control optimization and demand estimation under various traffic scenarios. Future work may include the extension of the road toll design problem to consider multiple modes (e.g., private vehicles and trucks), the dynamics of information acquisition and learning processes of users and long-term transport infrastructure investment decisions in a multi-period planning framework.

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