EVOLUTIONARY GAME-THEORETIC MODEL FOR PERFORMANCE RELIABILITY ASSESSMENT OF ROAD NETWORKS

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ABSTRACT

Reliability is recognized as an essential element for the planning and design of contemporary transportation systems. This paper provides a discrete-time evolutionary game-theoretic model incorporating day-to-day adjustment (route choice) behavior of users under the potential influence of perturbations (degradation-inducing events) for assessing the reliability of road networks. The proposed methodology is implemented on a test network with congestion effects. The results signify the importance of considering users' evolutionary process with 'memory' and taking into account transient states to assess the performance reliability as the system evolves over time.

INTRODUCTION

Transportation systems performance is subject to degradation caused by a variety of sudden changes or (un)expected events, such as accidents, adverse weather conditions and natural disasters, whose effect may be persistent over time. The evolution of the degraded system performance is influenced by dynamic adjustment mechanisms employed by users in order to respond to the new network traffic flow conditions. Despite the importance of considering this dynamic interplay between system performance and users' behavior processes for measuring the reliability of transportation networks, current approaches are static, in the sense that they rely on assumptions of time-invariant attributes in users' behavior. Section 2 describes the system attributes whose behavior is considered to be stochastic and Section 3 presents the evolutionary game-theoretic model and a simulation procedure with which is combined. Section 4 provides the experimental setup of the study and the results obtained from model implementation, and Section 5 concludes.

RELIABILITY DETERMINANTS OF ROAD NETWORKS

A variety of transportation systems have been investigated with regard to reliability considerations, ranging from urban road to freight networks. Despite the importance of reliability in the planning and design of transportation networks, the assessment of its various metrics is considered as a complex issue, due to the fact that it is influenced by many different components of the system. The complexity of the task increases because the forces that make transportation systems instable and unreliable relate to phenomena of uncertain and stochastic nature, whose attributes are difficult

to model or predict. By and large, reliability depicts the capability of the transportation system to adequately respond to alternative (expected or unexpected) events and its metrics corresponds to probabilistic values able to provide a distinction between alternative system states. In the transportation systems analysis, two dimensions of reliability have been identified, namely, the connectivity and the performance reliability (Bell and Iida 1997, Bell 2000). The former relates to system failures that could potentially lead to the loss of connection among trip origins and destinations, while the latter relates to the system ability to provide a certain level-of-service (LoS).

As a metric of network performance reliability, the travel time reliability has been widely used because it can depict the stability of the system. Also, other alternative perspectives of performance reliability have been investigated, such as network capacity reliability (Du and Nicholson 1997, Chen et al. 2002) and network capacity flexibility (Morlok and Chang 2004). The current study investigates the assessment of the total travel time reliability, since such a metric provides information for the state of the complete system. The specific reliability measure is defined as the probability of the total network travel time to be less than a pre-specified threshold. Also, results regarding the path travel time reliability are obtained, as being a by-product of the evaluation of total travel time. The measurement of the performance reliability in this study is based on the modeling of the stochastic nature of the various attributes of the system and the estimation of the users' responses to the various system states. The system examined here refers to a transportation network with typical urban road characteristics and congestion effects. The attributes that compose a road network are the demand, the supply, the level of service and the users' characteristics.

Beginning with the demand side, although the characteristics of demand are usually considered as recurrent, fluctuations in travel demand constitute a common phenomenon in the operation of urban road networks (Stathopoulos and Karlaftis 2001). These fluctuations can be attributed to various sources, like seasonality, special events, the reaction of users to previously experienced network conditions (producing elasticity to the demand and departure time) and the stochasticity pertaining to the diurnal traffic demand. All of the above characteristics influence the scale of demand for transportation in urban networks. Fluctuations in the supply-side components also form a typical phenomenon in transportation systems. System supply in road networks is expressed through the network capacity. The level of network capacity is principally dependent on the capacity of the links, but it is further influenced by other, degradation-inducing factors, such as incidents, civil works taking place at certain sections of the network, the composition of traffic, the users' driving behavior, physical disasters and others.

In addition, the network level of service and, especially, travel time is affected by the most volatile and, hence, stochastic component of the system, which refers to users' characteristics. Travel time generally depends on the various system attributes mentioned previously, namely the demand, supply and users' characteristics. Particularly in the case of urban networks where traffic flow as well as the interaction among users (drivers) is increased, the link and path travel times are heavily dependent on the users' driving characteristics, which are distinguished by increased stochasticity and uncertainty. Another attribute that especially influences the state of urban road networks is the route choice procedure. In this procedure, the users actually react to the perceived road traffic conditions by making choices regarding their route from origins to destinations,

in accordance with the aggregate loading pattern in the network. The users typically tend to make their choices collectively (as a group) in an attempt to minimize their perceived travel cost, usually expressed in terms of travel time.

When no measurements are available regarding the network operation, the performance reliability of a system composed of stochastic components such as those mentioned previously can be based on classic methods employed in engineering systems, like stochastic modeling and mathematical simulation. In this way, all system attributes with stochastic nature are modeled as random variables which follow estimated statistical distributions, while the results are drawn after performing an adequate number of runs. The scheme which is mostly preferred in simulating the operation of road networks for reliability assessment involves the performance of a number of alterations to the network and, subsequently, the estimation of the users' responses through a traffic assignment procedure.

Nonetheless, in realistic urban networks where most trips are taking place on a repetitive (usually, day-by-day) basis, the choices of users do not rely on the actual traffic conditions in the network, but rather they evolve over time with respect to the experienced traffic conditions, which are conditional to past travel behavior. Namely, the route choice is based on the historical rather than the actual travel cost. Also, it is evident that although the trajectory of the state of urban road networks moves towards equilibrium, it rarely equilibrates in practice. This is because the traffic conditions required to ensure the system stability cannot be met in real-world situations. In order to take into consideration the daily variation of the various system attributes, the system reliability is examined here within a discrete-time dynamic (evolutionary) manner. The next section describes a model able to represent users' reactions to the stochastic variations in the system attributes in an evolutionary fashion.

METHODOLOGICAL APPROACH OF THE STUDY

The simulation of the route choice procedure constitutes an integral part of the planning and design of urban road networks. Traditionally, the formulation of the route choice problem follows an equilibrium assumption and various well documented mathematical programming models have been proposed for estimating the network loading pattern (Sheffi 1985, Bell and Iida 1997). In the existing literature of reliability assessment in road networks, the estimation of the users' response to the fluctuations of the system attributes is based on models of static (single pass) assignment. However, these models are inadequate to simulate the users' memory and learning process, and they are concentrated on the mean values of the system attributes. Moreover, static equilibrium models cannot simulate transient states, as emerging from modifications in different attributes of the system, and they do not perform statistical analysis of the system performance during transient periods (Cantarella and Cascetta 1995).

Alternative models have been proposed to represent the dynamic route choice mechanisms of users. The inter-periodic or day-to-day models rely on dynamic control strategies, where the attractor is typically the equilibrium point. There is a distinction among the models used in the dynamic route choice procedure, in terms of whether the system is considered as continuous-time or discrete-time. Models of continuous-time are based on the theory of ordinary differential equations (Smith 1982, Friesz et al. 1994, Nagurney and Zhang 1996, Watling 1999). On the other hand, discrete-time models are based on Markovian learning procedures

(Cascetta 1989, Cantarella and Cascetta 1995, Jha et al. 1998, Watling 2002a, Watling 2002b, Yang 2004).

In the current study, a discrete-time model based on the evolutionary game theory (Smith 1982, Weibull 1995) is adopted in order to represent the dynamic (day-to-day) route choice procedure, where users' reaction is based on the assumption of bounded rationality. In particular, the evolution of route preferences is modeled through a stochastic discrete choice model of the logit logic, which enables to 'capture' the possibility that users make imperfect (sub-optimal) choices, as described within the framework of bounded rationality. The modeling process provides an extension of the classical non-cooperative game-theoretic formulation of the static user equilibrium assignment to a repeated (evolutionary) game with 'memory'. In this model, the current choice of each player (user) is made with respect to the history of the path travel times as resulted by the choices made by all players in the past. Briefly, taking $F_p(x_t)$ to be the cost of path $p \in \Re$ when it is traversed by flow x_t connecting the nth origin-destination pair during time t, the probability that path p is chosen can be given as:

$$\pi_p(x_t) = \frac{\exp(-\theta F_p(x_{t-1}))}{\sum_{q \in \Re} \exp(-\theta F_q(x_{t-1}))} \tag{1}$$

where θ is the route choice parameter. In turn, the logit evolutionary process can be expressed as a discrete-time dynamical system in the following form:

$$x_t - x_{t-1} = u_p(x_{t-1})\left(d\,\pi_p(x_t) - x_{t-1}\right) + \left(1 - u_{t-1}\right)x_{t-1} \tag{2}$$

where d is the travel demand between the nth origin-destination pair and u_p represents the percentage of users that will reconsider their route on day t. In the case where $u_p(x_t) = 1$ (i.e. all users will reconsider their route on day t), the dynamical system evolution reduces to the form:

$$x_t = d\,\pi_p(x_t) \tag{3}$$

It is straightforward to show that the fixed point of the above dynamical system is the stochastic user equilibrium.

Equation (2) describes a stochastic process of an equilibrium-tending system and it can be used to simulate the route choice dynamics in an urban network. In order to utilize this system for reliability assessment purposes, it is suitably combined with a mechanism of simulating the disturbances of the other system attributes, i.e. the demand, link capacities and link travel times. Such a mechanism can be based on standard techniques of mathematical simulation. In the current study, a Latin Hypercube sampling method (Iman and Conover 1982) is preferred, since it enables the production of random numbers from pre-specified (multinomial) distributions, which are used here to represent the daily variations of the demand, supply and link travel time for a certain period. Then, the route choice dynamics are estimated through the above evolutionary process. The next section presents empirical results of the application of the proposed model on a test network.

COMPUTATIONAL EXPERIMENTS AND RESULTS

The proposed methodology is implemented here on an extensively used (Cantarella and Cascetta 1995, Bell 2001, Yang 2004) test network, in order to gain insight about its numerical properties. The present 3×3 node grid network is composed of 12 links giving rise to 6 paths servicing a single origin-destination pair (Figure 1).
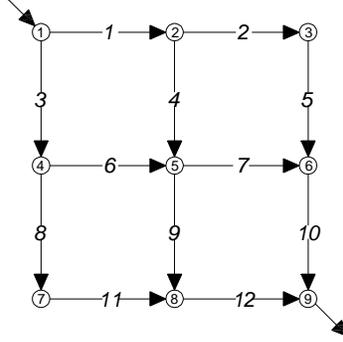

Figure 1: Configuration of the test network

The mean value of the demand is taken equal to $d = 500$ vehicles per hour (veh/hr). The travel cost is expressed by the travel time, which is estimated with the use of the Bureau of Public Works formula, as follows:

$$t_a = \frac{60L_a}{U_a}\left(1 + \beta\left(\frac{v_a}{L_a k_a I_a}\right)^m\right) \tag{4}$$

where t_a is the vehicular travel time in minutes (min) on link a, L_a is the link length in kilometers (km), U_a is the link free-flow speed in kilometers per hour (km/hr), β and m are positive constants depending on the operational characteristics of the network, v_a is the flow on that link in veh/hr, k_a is the link jam density in vehicles per kilometer per lane (veh/km), and l_a is the number of link lanes. The present example allows congestion effects by using the values $L_a = 1\,km$, $U_a = 50\,km/hr$, $k_a = 200\,veh/km$, $l_a = 1$ lane for all links, $\beta = 0.15$ and $m = 3$.

One of the most crucial aspects of the logit evolutionary system described in equation (2) is parameter θ which represents the dispersion (spread) in the users' cost perception. Relatively small θ values stand for larger dispersion in users' perceived cost and vice-versa. It has been shown (Yang 2004) that large θ values can expel the model from converging to the fixed point. Hence, the current study adopts a relatively small θ value ($\theta = 1$). Also, a model with 'short memory' has been preferred wherein all users reconsider their route choice every day with respect to the cost experienced on the previous day, i.e. $u_p = 1$ as described in equation (3). Table 1 presents the paths traversing the test network. The assessment of the performance reliability in the current network spans over a total period of 500 days (≈ 1.5 year). An initial investigation of the model performance is first carried out by applying the logit evolutionary system for a limited number of 50 days. Starting from an arbitrary point of path flows, it is found that it takes almost 15 days for the system to reach the fixed point, through converging to the vector x^* of user-equilibrium path flows

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$(x_1^*, x_2^*, x_3^*, x_4^*, x_5^*, x_6^*) = (89, 80.50, 80.50, 80.50, 80.50, 89)$ veh/hr, as it is shown in Figure 2(*a*), with an associated vector τ^* of path travel times $(\tau_1^*, \tau_2^*, \tau_3^*, \tau_4^*, \tau_5^*, \tau_6^*) =$ $(5.22, 5.32, 5.32, 5.32, 5.32, 5.22)$ min and total travel time equal to 2642 veh-min.

Table 1: Description of the Test Network Paths

Path No	Links Composing Paths
1	1-2-5-10
2	1-4-7-10
3	1-4-9-12
4	3-6-7-10
5	3-6-9-12
6	3-8-11-12

In order to demonstrate the system evolution under disturbance, a shock (sudden change) is imposed to the system, i.e. the reduction of the link #7 free-flow speed from $50\,km/hr$ to $25\,km/hr$ on day No 22 to day No 25. During the shock period, a considerable portion of users who had selected the paths including link #7, i.e. path No 2 and No 4, divert to the other four paths (see Figure 2(*b*)). The system is found to shortly return to the stochastic equilibrium point after about 4 days. This short response period can provide an evidence of the resilience of the given system.

In order to perform the reliability assessment, demand, link capacity and free-flow travel time are treated as random variables following normal distributions. In particular, the demand is simulated as a normal $N(d, 0.1 \times d)$ variable, where d is the nominal demand value ($d = 500$ veh/hr), with variance taken equal to the arbitrary 10% of the mean. In the same fashion, link capacities and free-flow speeds are modeled as $N(C_a, 0.1 \times C_a)$ and $N(U_a, 0.1 \times U_a)$ respectively. Thus, a random number from those distributions is used in each day in order to represent the system state concerning the demand, supply and link travel time. It should be noted that a variance of size 10% of the nominal values of each stochastic variable on a day-to-day basis gives rise to a highly stochastic system.

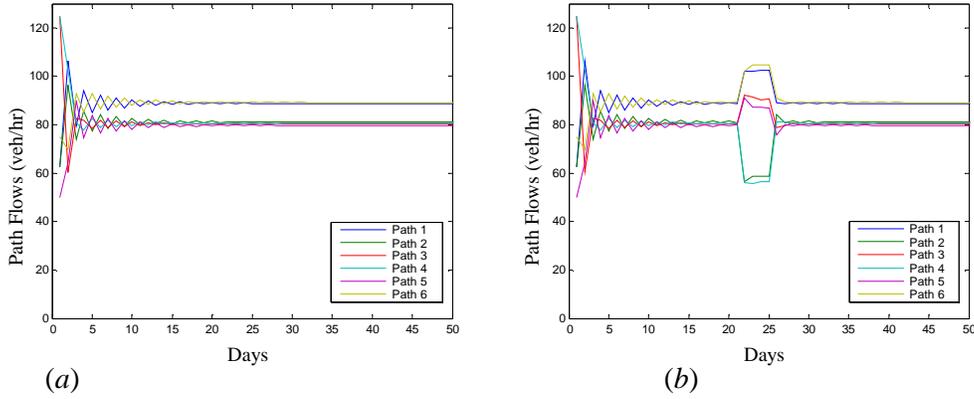(*a*)   (*b*)

Figure 2: Temporal evolution of the test network path flows to convergence

Despite that in the current study the values of the various system attributes are considered to be uncorrelated, in reality fluctuations among different components of

the system are usually correlated. The correlations among different system attributes can be included in the Latin Hypercube sampling procedure, since this method allows such modeling extensions. However, the analysis of correlations among the different components of the system goes beyond the scope of the current study. Moreover, bearing in mind that the reliability assessment here stands for cases of purely random perturbations in the system attributes, the assumption of normality cannot be considered as an oversimplification in this study.
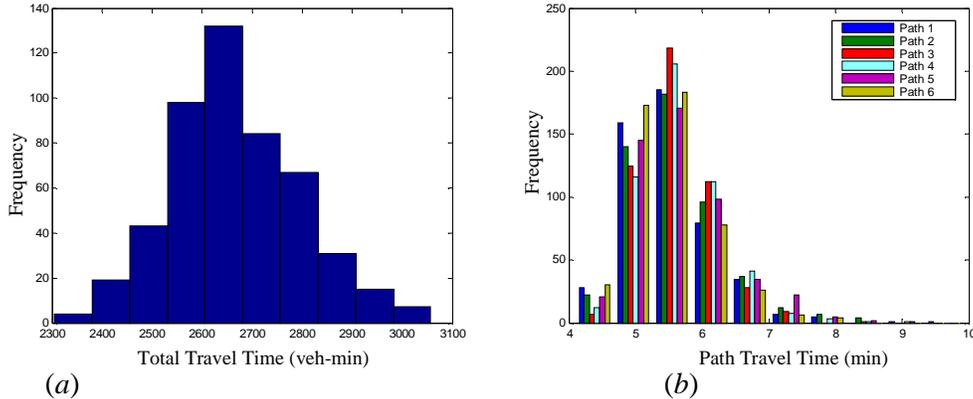
(a) (b)

Figure 3: Illustration of the distribution of (a) total travel time and (b) path travel times over the study horizon

Based on the distributions of total travel time (see Figure 3(a)) and path travel time (see Figure 3(b)) over the studying horizon of the 500 days, it can be derived that their mean values basically coincide with those values found at equilibrium conditions, as they were estimated in the initial investigation of the model. This outcome reveals a latent 'collective order' regarding the response of users to the system component perturbations. Thus, despite that the system is subject to significant degradation and the increased stochasticity in the behavior of all components, it appears to follow a predictable evolution pattern through absorbing perturbations around the fixed point, i.e. the stochastic user equilibrium state, in the long run.

CONCLUSIONS

The assessment of the performance reliability of a road transportation network constitutes an increasingly important issue for measuring and improving the quality of services delivered to the users, including both passengers and freight carriers. Existing approaches for reliability assessment are typically based on assuming time-invariant users' behavior responses to perturbation (degradation-inducing events) in the various system components. This paper provides a stochastic methodological framework for considering the day-to-day evolutionary behavior of users in the performance reliability assessment of road networks. The route choice dynamics is represented here as a repeated (evolutionary) game with 'memory', through a logit evolutionary system, whose fixed point refers to the stochastic user equilibrium. This model is combined with a mathematical simulation technique for simulating the disturbances of travel demand, link capacities and link travel times.

The proposed methodology is implemented on a test network with congestion effects. The results signify the importance of considering users' evolutionary process with 'memory' and taking into account transient states to assess the performance

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reliability as the system evolves over time. In particular, the solutions obtained from the long-term evolution of travel costs and path flows were found to be concentrated around that of the stochastic user equilibrium state. This output can be possibly attributed to the response of the collective behavior of users in urban networks so that absorb perturbations in different components of the system. The methodology can also provide insight about how time-point changes at the micro (link)-level can cause long-term changes in the system-wide operational performance.

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